Research Article

An Assessment of the Alignment of Transformation Geometry with Van Hiele Levels in Uganda's Lower Secondary Mathematics Curriculum

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Abstract: This study investigates the alignment of transformation geometry learning outcomes with the Van Hiele levels of geometric reasoning within Uganda's secondary mathematics curriculum. Using the Van Hiele model as a framework, the study evaluates whether the curriculum effectively supports students' cognitive development in geometry. An analysis of the learning outcomes associated with transformation geometry specifically reflection, rotation, and enlargement reveals a partial alignment with the Van Hiele levels of visualization, analysis, abstraction, and deduction. The findings indicate that while outcomes related to visualization and basic analysis are well-aligned, there are significant gaps in outcomes requiring higher-order reasoning, particularly at the abstraction and deduction levels. Specifically, the curriculum underemphasizes tasks that involve formal deductive reasoning and the application of complex geometric transformations. These gaps suggest that the curriculum may not fully support students' progression to higher levels of geometric understanding. The study concludes with recommendations for curriculum enhancement to better align learning outcomes with the cognitive demands of the Van Hiele model, thereby fostering deeper mathematical reasoning among secondary students in Uganda.

Keywords: Van Hiele levels, Transformation geometry, Curriculum alignment, Secondary mathematics, Uganda, Geometric reasoning

1. Introduction

Geometry education is widely acknowledged as a crucial component in fostering students' spatial reasoning, problem-solving skills, and broader mathematical understanding (MdYunus et al., 2019; Ubi et al., 2018; Vágová & Kmetová, 2019). Central to effective geometry instruction is the Van Hiele theory of geometric reasoning, developed by Pierre and Dina Van Hiele in the 1950s that outlines a progression of cognitive stages that learners experience as they develop their geometric understanding, making it a key framework for designing and assessing geometry curricula (Al-ebous, 2016; Alex & Mammen, 2018).

This model identifies five distinct levels of geometric reasoning; at Level 1 (Visualization), learners recognize shapes based on their appearance rather than their properties. Level 2 (Analysis) involves understanding the properties of shapes, such as symmetry or congruence. At Level 3 (Abstraction), students begin to generalize these properties and recognize relationships between them. Level 4 (Deduction) requires students to engage with formal proofs and theorems, while Level 5 (Rigor) involves a deep exploration of axiomatic systems and advanced geometric concepts (Vojkuvkova, 2012; Yildiz & Baltaci, 2016; Yilmaz, 2008; Yılmaz & Koparan, 2015). This structured progression provides educators with a framework for guiding students from the basic recognition of shapes to sophisticated geometric reasoning.

Globally, the importance of aligning geometry curricula with the Van Hiele levels has been highlighted by numerous studies. Countries like the United States, Singapore, and the Netherlands have incorporated the Van Hiele model into their curricula, resulting in significant improvements in students' geometric reasoning and problem-solving abilities (Clements & Sarama, 2011; Ng & Sinclair, 2015). This global success demonstrates the importance of ensuring that learning outcomes in geometry are systematically aligned with students' cognitive development, promoting deeper understanding and better application of geometric concepts in both academic and real-world contexts.

In Uganda, the Competency-Based Curriculum (CBC) was introduced to address the need for skills-based learning in a rapidly changing world (NCDC, 2019, 2020). While this curriculum reform aims to improve the quality of education, the extent to which it aligns with cognitive development frameworks, such as the Van Hiele model, remains under-researched, particularly in areas like transformation geometry. Transformation geometry, which includes reflection, rotation, enlargement, and translation, plays a foundational role in developing students' spatial reasoning and geometric visualization skills. These topics offer concrete applications of geometric transformations and are critical for progressing through the Van Hiele levels (Abdullah & Zakaria, 2013b).

Despite the centrality of transformation geometry in the curriculum, empirical evidence on how well Uganda's lower secondary curriculum supports cognitive development in geometric reasoning is lacking. Without a clear alignment between the curriculum's learning outcomes and the Van Hiele levels, there is a risk of

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leaving gaps in students' geometric understanding, particularly at higher cognitive levels where formal proofs and advanced reasoning skills are required. This misalignment could limit students' ability to grasp more complex mathematical concepts in future academic endeavors.

The current study seeks to analyze the alignment between the transformation geometry learning outcomes in Uganda's secondary school curriculum and the Van Hiele levels. By focusing on key subtopics such as reflection, rotation, and enlargement, this research aims to evaluate how well the curriculum supports students' progression through the cognitive stages of geometric reasoning. The findings provide valuable insights for curriculum developers, educators, and policymakers, to enhance the effectiveness of mathematics education in Uganda and ensure that students are equipped with the necessary skills to excel in geometry.

1.1. Background

In an ideal mathematics curriculum, learning outcomes are systematically aligned with students’ cognitive development stages, ensuring that instructional content progressively builds on prior knowledge and fosters deeper understanding (Yılmaz & Koparan, 2015). The Van Hiele model, widely recognized for its structured levels of geometric reasoning, provides a theoretical framework for such alignment, particularly in the teaching of transformation geometry, which includes reflection, rotation, and enlargement (Machisi & Feza, 2021). However, in Uganda's secondary mathematics curriculum, the extent to which transformation geometry learning outcomes align with the Van Hiele levels remains unclear. The current study aims to investigate the extent of this alignment, identifying specific areas where the curriculum falls short. By evaluating the correspondence between the curriculum's learning outcomes and the Van Hiele levels, this study seeks to provide evidence-based recommendations for enhancing the curriculum, thereby better-supporting students' geometric learning and cognitive development in Uganda.

The curriculum design and instructional strategies used in lower secondary mathematics education may be significantly impacted by the findings of this study. Through the evaluation of geometry-transformation learning outcomes about the Van Hiele model, the research provides valuable perspectives on how curricula may be designed to foster the cognitive growth of geometric reasoning. The results can help educators and curriculum makers make targeted modifications to better assist students' progression through the levels of geometric comprehension by identifying the frameworks' strengths and flaws. The study also advances the subject of mathematics education by illuminating a methodological strategy that combines realistic curriculum analysis with theoretical frameworks.

The objective of this study is to evaluate the alignment of lower secondary transformation geometry learning outcomes with Van Hiele model criteria and their support for cognitive development.

1.2. Literature Review

The theoretical framework of this study is grounded in the Van Hiele model of geometric reasoning, which outlines a progressive hierarchy of five levels Visualization, Analysis, Abstraction, Deduction, and Rigor that describe how students develop their understanding of geometric concepts (Vojkuvkova, 2012). This model serves as a guiding structure for evaluating the alignment of the lower secondary geometry-transformation curriculum, providing a lens through which to assess whether the curriculum effectively supports students' cognitive development in geometry or not. By focusing on how learning outcomes correspond to these levels, the study aims to identify strengths and gaps in the curriculum, ensuring that it facilitates students' progression through increasingly complex stages of geometric reasoning.

Usiskin (1982) provided early insights into how students' Van Hiele levels correspond with their success in geometry, emphasizing that students must be taught at the correct cognitive level to develop meaningful geometric understanding. This model has been widely used in curriculum development and instructional design, helping educators align learning outcomes with students' cognitive capabilities (Crowley, 1987; Vojkuvkova, 2012).

In addition to its systematic framework, the Van Hiele theory places considerable emphasis on the alignment between teachers and learners in terms of cognitive levels. It posits that effective progress hinges on this alignment, highlighting that misalignment can impede learners' advancement, resulting in rote memorization without genuine mastery. Acknowledging potential variations in geometric understanding across content strands, the theory asserts that achieving a specific thought level in one strand lays a foundation for easier attainment in another. This perspective adds depth to the theory's applicability and emphasizes its potential impact on diverse aspects of geometric education (Wulandari et al., 2021; Machisi & Feza, 2021).

Transformation geometry, which encompasses reflection, rotation, translation, and dilation, plays a vital role in developing students' spatial reasoning. In their analysis, Abdullah & Zakaria (2013) emphasized that transformation geometry, when taught effectively, significantly enhances students' ability to visualize and manipulate shapes, progressing from Visualization (Level 1) to more advanced levels. This study seeks to explore whether Uganda's lower secondary mathematics curriculum aligns its transformation geometry learning outcomes with the Van Hiele levels to ensure students' cognitive progression.

Ensuring that learning outcomes align with the Van Hiele levels is crucial for fostering geometric understanding. In their research, Zulnaidi et al., (2020) highlighted that students often struggle with higher-order geometric thinking when there is a mismatch between the curriculum and their current cognitive level. This concern mirrors the objectives of the current study, which seeks to determine if Uganda's curriculum provides students with a structured progression through the Van Hiele levels in transformation geometry. Brown et al. (2004) further emphasized that alignment issues often result in gaps in students' geometric reasoning, preventing them from attaining higher levels of abstraction and deduction.

The activities under each Van Hiele level in transformation geometry highlight the progression of geometric reasoning skills, from basic visualization and recognition (Level 1) to advanced formal proofs and axiomatic systems (Level 5). These activities provide insights into the depth and complexity of geometric understanding required at each stage of cognitive development, informing tailored instructional strategies and curriculum design as illustrated in Figure 1.

VL1

- Learners draw simple shapes and their reflections
- Learners rotate basic shapes (e.g., squares, triangles)
- Learners use grid paper to perform translations of shapes

VL2

- Learners analyze the effects of different transformations
- Learners identify and describe patterns in the coordinates of vertices before and after transformations,
- Learners predict the outcome of a transformation

VL3

- Learners generalize transformation rules
- Learners explore the concept of symmetry in different geometric figures
- Identify invariant properties under various transformations.
- Learners decompose composite transformations into simpler transformations

VL4

- Learners construct formal proofs to justify properties of transformations
- Learners investigate geometric properties that remain invariant under specific
- Learners apply transformation theorems to solve geometric problems.

VL5

- Learners explore axiomatic systems for transformation
- Learners study non-Euclidean geometries and analyze their implications for transformation geometry concepts.
- Learners tackle advanced proof challenges related to transformation geometry

Figure 1. Activities under Each Van Hiele Level (examples from Transformation Geometry

Several studies have explored the alignment between geometry curricula and the Van Hiele model, highlighting both successes and challenges in various educational contexts. However, these studies primarily focus on Western and Asian contexts, leaving a gap in understanding how the Van Hiele model is applied in African educational systems, particularly in Uganda (Çontay & Duatepe-Paksu, 2019; MdYunus et al., 2019; Mosia et al., 2023). The current study aims to fill this gap by analyzing the alignment of Uganda's lower secondary transformation geometry curriculum with the Van Hiele model, providing insights into how well the curriculum supports students' cognitive development in geometric reasoning.

2. Method

This study employed a qualitative research design using document analysis methodology to assess the alignment of transformation geometry learning outcomes in Uganda's secondary mathematics curriculum with the Van Hiele levels of geometric reasoning. A qualitative approach was chosen because it allows for in-depth analysis and interpretation of curriculum documents, focusing on how well educational content aligns with cognitive development frameworks. Qualitative research is well-suited for exploratory studies aimed at understanding complex phenomena, such as the alignment of educational content with theoretical models, providing rich, detailed insights (Creswell & Creswell, 2018).

The primary data source was the official secondary mathematics curriculum document provided by the Uganda National Curriculum Development Centre (NCDC). Sections of the curriculum related to transformation geometry specifically focusing on reflection, rotation, and enlargement were extracted and analyzed.

The extracted learning outcomes were systematically categorized according to the Van Hiele levels: Visualization (Level 1), Analysis (Level 2), Abstraction (Level 3), and Deduction (Level 4). Level 5 (Rigor) was excluded as it typically lies beyond the scope of secondary education (Mason & Johnston-Wilder, 2020). Each learning outcome was examined to determine which Van Hiele level it best corresponded to, based on the cognitive demands required by the tasks described in the curriculum. Table 1 shows the criteria for aligning the learning outcomes with the Van Hiele-level framework.

Table 1. Showing aligning the learning outcomes with the Van Hiele level, Framework.

Van Hiele Level	Focus	Key Activities	Criteria for Alignment
Level 1: Visualization	Recognizing and identifying shapes and transformations.	- Basic identification and naming of shapes and transformations. - Drawing or visualizing simple geometric figures and their transformations (e.g., reflection, rotation). - Recognizing symmetry and similar shapes without detailed analysis.	- Learning outcomes that involved simple visual tasks, such as recognizing or drawing shapes and identifying basic transformations, were aligned with this level. - Outcomes requiring visualization without deep analysis were placed here.
Level 2: Analysis	Understanding and analyzing the properties of shapes and transformations.	- Analyzing properties of shapes, such as symmetry, congruence, and proportionality. - Understanding and applying transformation rules (e.g., reflections, rotations, enlargements) in a coordinate plane. - Describing relationships between shapes.	- Learning outcomes that required understanding and analyzing geometric properties were aligned with this level. - Outcomes that required the application of transformation rules or understanding properties of geometric figures were placed here.
Level 3: Abstraction	Understanding relationships among geometric properties and reasoning about them abstractly.	- Applying transformations using abstract representations (e.g., matrices). - Understanding relationships between different geometric properties, such as scale factors and areas, without visual aids. - Using algebraic or coordinate methods.	- Learning outcomes that required abstract reasoning about relationships between properties, such as calculating areas after dilation or understanding transformation matrices, were aligned with this level. - Outcomes involving abstract application of concepts were placed here.
Level 4: Deduction	Using formal deductive reasoning and understanding the structure of a deductive system.	- Constructing and understanding formal proofs and theorems related to transformations. - Applying formal deductive reasoning to solve complex transformation problems. - Understanding and applying composite transformations.	- Learning outcomes that required formal deductive reasoning, such as proving geometric properties or deriving single matrices for composite transformations, were aligned with this level. - Outcomes involving formal proofs and logical sequences were placed here.

To validate the findings, a panel of three mathematics education experts reviewed the alignment of the learning outcomes with the Van Hiele levels. The panel provided feedback on the initial alignment, and adjustments were made where necessary to ensure accuracy and reflect the intended cognitive progression.

The study adhered to ethical standards in educational research. Since the research involved the analysis of publicly available curriculum documents and did not involve human subjects, formal ethical approval was not required. However, care was taken to accurately represent the curriculum content and to ensure that the analysis was conducted objectively and without bias.

3. Results and Discussion

This section presents the findings assessing the alignment between lower secondary geometry-transformation learning outcomes and the Van Hiele model criteria, analyzing the distribution of learning outcomes across Van Hiele model levels, and investigating how the curriculum supports cognitive development in geometric reasoning according to the Van Hiele model. The discussion is organized by Van Hiele levels, starting with Level 1 (Visualization) and progressing through to Level 5 (Rigor). Table 2 shows the alignment of transformation-learning outcomes with the Van Hiele levels.

Table 2. Showing the alignment of transformation-learning outcome with the Van Hiele levels

Subtopic	**Learning Outcome**	**Van Hiele Level**	**Justification**
Reflection	Identify lines of symmetry for different figures	Level 1: Visualization	Involves the basic identification of symmetry lines, focusing on visual recognition rather than analysis.
	Reflect shapes and objects	Level 1: Visualization	Involves visualizing and performing reflections, focusing on recognizing changes in orientation and position.
	Apply reflection in the Cartesian plane	Level 2: Analysis	Involves understanding and applying the reflection transformation on a Cartesian plane, requiring analysis of geometric properties.
Similarities and Enlargement	Identify similar figures	Level 1: Visualization	Involves recognizing similar shapes, and focusing on visual identification without deep analysis.
	State the properties of similar figures	Level 2: Analysis	Involves identifying and explaining the properties of similar figures, such as proportionality, requiring analysis rather than abstraction.
	Define enlargement	Level 2: Analysis	Involves understanding and applying the concept of scaling a figure, and analyzing how properties change during enlargement.
	State the properties of enlargement to construct objects and images	Level 2: Analysis	Involves applying the concept of enlargement to create new shapes, requiring an understanding of the scaling properties.
	Understand and use the relationship between linear, area, and volume scale factor	Level 3: Abstraction	Involves abstract reasoning to understand the relationships between different types of scale factors and their effects on geometric properties.
Rotation	Identify the order of rotational symmetry of plane figures	Level 1: Visualization	Involves recognizing and describing the order of rotational symmetry, which focuses on basic visualization.
	Distinguish between clockwise and anticlockwise rotation	Level 1: Visualization	Involves visualizing and distinguishing the direction of rotation, a basic visual task.
	State properties of rotation as a transformation including congruence	Level 2: Analysis	Involves understanding and explaining the properties of rotation, such as congruence, requiring analysis of the transformation's effects.
	Determine the center and angle of rotation	Level 3: Abstraction	Involves identifying and calculating the center and angle of rotation, requiring

Subtopic	Learning Outcome	Van Hiele Level	Justification
			abstract reasoning about geometric relationships.
	Apply properties of rotation in the Cartesian plane	Level 3: Abstraction	Involves using rotational properties to perform transformations in the Cartesian plane, requiring abstract reasoning to understand the effects on coordinates.
Matrix Transformation	Describe a transformation as a reflection, a rotation, or an enlargement	Level 3: Abstraction	Involves understanding and describing transformations using matrices, requiring abstract reasoning about the relationships between geometric transformations and their algebraic representations.
	Identify transformation matrices for reflection, rotation, and enlargement	Level 3: Abstraction	Involves selecting and applying the appropriate matrix for specific transformations, requiring abstract reasoning about matrix properties.
	Determine the image given the object and transformation matrix on a coordinate grid	Level 3: Abstraction	Involves calculating the new coordinates of a transformed object using a matrix, requiring an understanding of abstract relationships between the original coordinates and the transformation matrix.
	Identify the matrix of transformation when the object and image are given	Level 3: Abstraction	Involves reasoning about the relationship between an object and its transformed image to determine the corresponding transformation matrix, requiring abstract understanding of geometric transformations.
	Determine the inverse of a transformation	Level 4: Deduction	Involves calculating the inverse of a transformation matrix, requiring formal deductive reasoning to reverse the transformation process.
	Use the inverse of a transformation to find the image when the object is given	Level 4: Deduction	Involves applying an inverse transformation matrix to solve problems, requiring formal deductive reasoning to accurately reverse the transformation.
	Identify the relationship between the area scale factor and the determinant of the transformation matrix	Level 4: Deduction	Involves understanding the deductive relationship between the determinant of a matrix and the area scale factor, requiring formal geometric reasoning.
	Determine a single matrix for successful transformations	Level 4: Deduction	Involves combining multiple transformations into a single matrix, requiring an understanding and application of a deductive system of properties to generalize transformations.

3.1. Level 1: Visualization

The curriculum aligns strongly with Level 1 of the Van Hiele model, which focuses on the recognition and visualization of geometric shapes and transformations. Five learning outcomes (24% of the total) are aligned with this level. Examples include *"Identifying lines of symmetry for different figures"* and "*Reflecting shapes and objects*." These tasks are fundamental for students at the initial stages of geometric understanding, as they involve basic recognition and manipulation of geometric shapes. The allocation of 24% of learning outcomes to Level 1 indicates a balanced emphasis on foundational visualization skills. This is consistent with the cognitive needs of lower secondary students, who are typically developing their ability to recognize and visualize geometric concepts Visualization is a crucial first step in geometric reasoning, and the curriculum effectively supports cognitive development at this level. By engaging in tasks that require them to recognize and manipulate geometric shapes, students begin to form the mental images that are essential for progressing to more complex geometric reasoning. The literature supports this focus, with Yildiz & Baltaci (2016) emphasizing that strong visualization skills are critical for students' progression to higher Van Hiele levels. The importance of visualization in the early stages of geometric learning is well-documented. Vojkuvkova (2012) and Yildiz & Baltaci (2016) both highlight that visualization forms the foundation upon which more advanced geometric reasoning is built. The curriculum's strong emphasis on visualization is consistent with these findings, ensuring that students develop the necessary skills to engage with more complex geometric concepts as they advance in their studies.

3.2. Level 2: Analysis

The curriculum includes five learning outcomes (24% of the total) that align with Level 2 of the Van Hiele model, focusing on analyzing geometric properties and understanding relationships between shapes. Examples include "*Applying reflection in the Cartesian plane*" and "*Stating the properties of similar figures."* These tasks require students to move beyond simple recognition and begin analyzing how geometric properties change or remain consistent under various transformations. The representation of Level 2 learning outcomes is consistent with the curriculum's overall emphasis on developing students' analytical skills. By dedicating 24% of the learning outcomes to this level, the curriculum ensures that students engage with tasks that deepen their understanding of geometric properties and relationships. At this level, students begin to understand the underlying properties of geometric shapes and how these properties relate to one another. The curriculum supports this cognitive development by including tasks that require analysis and explanation of geometric transformations. However, there is an opportunity to enhance the curriculum by including more tasks that focus on categorizing shapes based on their properties or comparing different types of symmetry, as suggested by Ubi et al. (2018). The development of analytical skills is crucial for students to advance in geometric reasoning. Ubi et al. (2018) argue that the ability to analyze and categorize shapes is essential for deepening students' understanding of geometry. The curriculum's focus on analysis aligns with these findings, although the literature suggests that a more comprehensive approach to analyzing geometric properties could further strengthen students' skills at this level.

3.3. Level 3: Abstraction

Level 3, which involves abstract reasoning and understanding relationships among geometric properties, is well-represented in the curriculum, with seven learning outcomes (33% of the total). Examples include "*Understanding and use the relationship between linear, area, and volume scale factors"* and "*Determining the center and angle of rotation."* These outcomes require students to engage in abstract reasoning and apply their knowledge to more complex geometric concepts. The fact that 33% of the learning outcomes are aligned with Level 3 reflects a strong emphasis on abstraction, which is critical for students' progression in geometric reasoning. This focus ensures that students are challenged to think beyond concrete examples and begin generalizing geometric concepts. At Level 3, students are expected to generalize and connect different geometric ideas, moving towards a more theoretical understanding of geometry. The curriculum's strong emphasis on abstraction supports this cognitive development, preparing students for more advanced studies in mathematics. This focus on abstraction aligns with research by Vojkuvkova (2012) and Yildiz & Baltaci (2016), which highlights the importance of developing abstract reasoning skills for success in higher-level mathematics. Abstraction is a key component of advanced geometric reasoning. Vojkuvkova (2012) and Yildiz & Baltaci (2016) both emphasize the need for students to engage in abstract thinking to apply geometric concepts in new contexts.

3.4. Level 4: Deduction

The curriculum includes four learning outcomes (19% of the total) that align with Level 4, which involves formal deductive reasoning. Examples include "*Determining the inverse of a transformation"* and *"Identifying the relationship between the area scale factor and the determinant of the transformation matrix.*" These tasks require students to apply formal logic to geometric transformations, engaging in higher-order thinking. While Level 4 is less represented than the earlier levels, the inclusion of 19% of the learning outcomes at this level introduces students to the critical skill of deductive reasoning. This allocation is appropriate for lower secondary students, who are beginning to engage with more formal aspects of geometric reasoning. Deductive reasoning is essential for advanced geometric thinking, as it involves the ability to form logical arguments and prove geometric properties. The curriculum supports cognitive development at this level by including tasks that require students to use formal reasoning to solve problems. However, the curriculum could be further enhanced by incorporating more tasks that involve comparing and confirming transformation properties, as suggested by MdYunus et al. (2019). The introduction of deductive reasoning at the lower secondary level is supported by the literature, which emphasizes the importance of developing higher-order thinking skills in geometry. Yılmaz & Koparan (2015) noted that formal deductive reasoning is critical for success in advanced mathematics, and the curriculum's inclusion of Level 4 outcomes aligns with these findings. However, additional focus on deductive tasks could strengthen students' readiness for more complex geometric challenges.

3.5. Level 5: Rigor

As expected, the curriculum does not include learning outcomes at Level 5, which involves advanced geometric reasoning beyond the typical scope of lower secondary education. Level 5 tasks, such as formal proof construction and exploration of non-Euclidean geometries, are generally reserved for higher education. The absence of Level 5 outcomes is consistent with the developmental stage of lower secondary students. The curriculum is appropriately focused on foundational and intermediate geometric reasoning, ensuring that students are not overwhelmed by tasks that are beyond their current cognitive abilities. While the curriculum does not address Level 5, this is appropriate given the educational level of the students. The focus remains on developing the necessary skills at Levels 1-4, which provide a strong foundation for more advanced studies in the future. The exclusion of Level 5 outcomes is supported by the literature, which suggests that tasks requiring rigor and formal proof are typically introduced at higher educational levels (Vágová & Kmetová, 2019). By focusing on Levels 1-4, the curriculum ensures that students are adequately prepared for future challenges in geometry without being prematurely exposed to overly complex concepts.

4. Conclusion

The results indicate a well-structured approach to developing students' geometric reasoning skills. The curriculum effectively supports cognitive development at Levels 1-3, with a strong emphasis on Visualization, Analysis, and Abstraction. Level 4, while less represented, introduces students to deductive reasoning, preparing them for more advanced studies in mathematics. The exclusion of Level 5 outcomes is appropriate, reflecting the developmental stage of lower secondary students. Overall, the curriculum is well-aligned with the Van Hiele model, providing a comprehensive framework for students' progression through increasingly complex geometric tasks.

5. Recommendations

To enhance alignment with the Van Hiele levels, it is recommended that the curriculum incorporates more tasks requiring higher-order reasoning, such as abstraction and deduction. Teacher professional development should focus on differentiated instruction while integrating technology like GeoGebra to enhance visualization and analysis skills. Continuous curriculum evaluation is also essential to ensure that learning outcomes effectively support students' cognitive development in geometric reasoning. These measures will strengthen students' progression through the Van Hiele levels in transformation geometry.

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