Research Article

Exploring High-achieving Eighth Grade Students' Solution Strategies and Performance on the Arithmetic Mean Problems

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Article History: Received: 23 July 2024; Accepted: 3 December 2024; Published online: 31 December 2024

Abstract: Arithmetic mean is a concept that has many uses in daily life (such as; meteorology, medicine, and agriculture). Arithmetic mean is a concept that has mathematical and statistical aspects, so both aspects must be known in depth in order to understand the arithmetic mean conceptually. A conceptual understanding of the arithmetic averaging algorithm can enable students to solve arithmetic averaging problems effectively and is the first step beyond understanding the statistical aspects of the concept. The aim of this research was to investigate high achievers' solution strategies and performance on the arithmetic mean problems. The participants consisted of 103 grade eighth students drawn from eight classes in five elementary schools of Yozgat, Turkey. All the participants were considered to be current achievers in mathematics, graded 4 or 5 out of 5, and selected via a purposive sampling method. Eighth-grade middle school students with high levels of achievement were given problem scenarios requiring flexible use of the arithmetic mean algorithm, and they were asked to solve the problems using all the strategies they knew. In addition, one student from each school was interviewed and the interview and written exam findings were compared and interpreted. It was found that nearly all high-achieving eighth grade students used the add-divide algorithm strategy. The number of students using multiple solution strategies is quite low. It was revealed that students' performance was lower in problems that required more than one term in a data set. The findings were supported by interviews with students.

Keywords: Arithmetic mean, High-achieving eighth grade students, Performance, Solution strategy

1. Introduction

Arithmetic mean is a concept that has many uses in daily life (such as; meteorology, medicine, and agriculture). For example, we may frequently encounter the concept of average while watching television or reading the newspaper (Zazkis, 2013; Güler, Gursoy, & Güven, 2016). One of the fundamental reasons for the difficulties students face is that they tend to think of the arithmetic mean concept solely as an algorithm (Watson, 2006; Marnich, 2008; Sirnik and Kmeti, 2010). Arithmetic mean is a concept that has mathematical and statistical aspects, so both aspects must be known in depth in order to understand the arithmetic mean conceptually (Cai, 1998, p.93). A conceptual understanding of the arithmetic averaging algorithm can enable students to solve arithmetic averaging problems effectively and is the first step beyond understanding the statistical aspects of the concept (Cai, 1995; Marnich, 2008).

In this study, eighth-grade middle school students with high levels of achievement (with math grades of 4 or 5 out of 5) were given problem scenarios requiring flexible use of the arithmetic mean algorithm (Cai, Lo, & Watanabe, 2002), and they were asked to solve the problems using all the strategies they knew. The problems used in the study were created based on previous research (Cai, Moyer, & Grochowski, 1999; Cai & Moyer, 1995; Uccellini, 1996; Konold & Pollatsek, 2002). In addition, interviews were conducted with the students. Findings were obtained about how the concept of arithmetic mean was taught by teachers and what kind of questions teachers included in the lesson.

How teachers teach a mathematical concept, what type of problems they use in their lessons, and the solution strategies they use when solving problems can affect students' solution strategies and performances. This study, unlike previous studies, was conducted on students with a grade of 4 or 5 in mathematics, and the strategies and performances of students in arithmetic mean problems were examined. In addition, interviews were conducted with students to determine the relationship between the strategies used by students and the teaching style of their teachers. In fact, high achieving students are expected to solve problems that require flexible use of the arithmetic mean algorithm with different solution strategies and reach correct results. Therefore, the results obtained from the study may allow us to make predictions about the effectiveness of teachers' measurement and evaluation practices. In line with the aforementioned objectives, the following sections will address understanding the concept of arithmetic mean, problem types and solution strategies related to the arithmetic mean algorithm.

1.1. Conceptual Knowledge of the Arithmetic Mean

Conceptual knowledge of the arithmetic mean can be considered in two categories, mathematical and statistical. Mathematically, conceptual knowledge includes an in-depth understanding of the mathematical

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Citation Information: Bütüner, S. Ö. (2024). Exploring high-achieving eighth grade students' solution strategies and performance on the arithmetic mean problems. *Turkish Journal of Mathematics Education*, *5*(3), 70-81.

operations in the add-divide strategy. Conceptual knowledge of the arithmetic mean as a statistical concept points to its representative role (Strauss & Bichler, 1988; Marnich, 2008).

For many students, the arithmetic mean is only seen as a mathematical concept and is associated solely with the add-divide algorithm. This is due to students being exposed to rote and procedural instruction during their school years (Mokros & Russell, 1995; Konold & Higgins, 2003; Russell & Mokros, 1996). The ideas of balance point and fair sharing are considered strong analogies in the conceptual understanding of the arithmetic mean (Bremigan, 2003; Uccelini, 1996; Van de Walle & Bay-Williams, 2013). Students who grasp these ideas tend to perform better in problems requiring flexible use of the add-divide algorithm (problems involving given arithmetic means, constructing datasets, or finding multiple terms) (Cai, 1998; Cai, 2000). A conceptual understanding of the arithmetic mean algorithm is important to understanding the statistical properties of the concept. Fair sharing and balance center strategies are presented below.

1.2. Fair Sharing and Center of Balance

Through the center of balance model in Figure 1, students can learn in depth the underlying meaning of the add-divide algorithm (Uccelini, 1996; Van de Walle, Karp & Bay-Williams, 2013). These models are also useful tools in showing students that the sum of the deviations from the mean, an important property of the arithmetic mean, is zero.

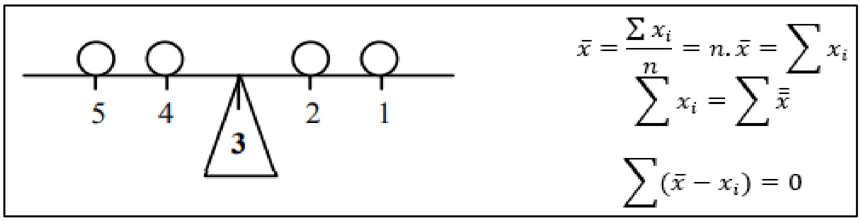


Figure 1. Arithmetic Mean Algorithm and Algebraic Manipulation (Marnich, 2008)

Arithmetic mean formula can be taught to students using a block stacking diagram (Van de Walle, Karp & Bay-Williams, 2013). When the block stacks on the left in Figure 2 are distributed until the levels are equal, the arithmetic mean of the data set is obtained. In the fair sharing model, It is possible to show that $\sum(x - x_i) = 0$.

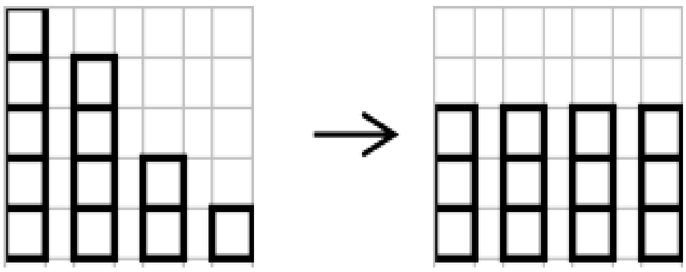


Figure 2. Fair Sharing Model [$\bar{x} = \frac{\sum x_i}{n} = \frac{4+3+2+1}{4}$ = 3; $\sum(\bar{x} - x_i) = (3 - 5) + (3 - 4) + (3 - 2) + (3 - 1) = 0$]]

1.3.Problems for the Use of the Arithmetic Mean Algorithm

Problems that require creating a data set with a certain arithmetic mean or finding more than one term should also be included in the courses. These types of problems are statistically more challenging, limit the use of the arithmetic mean algorithm, and guide students towards seeking conceptual solutions (Cai, 2000; Cai, Lo, & Watanabe, 2002; Leavy & O'Loughlin, 2006; Mokros & Russell, 1995). In the first phase of this study, students were provided with problems requiring flexible use of the arithmetic mean algorithm to identify their problem-solving strategies and their performance on these problems. The problems presented to the students in the study were formulated based on previous research and are provided in Appendix 1. Categorizations made by Cai et al. (2002) regarding problems that directly require the use of the add-divide algorithm and problem types requiring its flexible use are provided in Table 1. The next section discusses the strategies that can be employed to solve arithmetic mean problems.

Table 1. Theoretical Framework Used in the Classification of Problems (Cai et al, 2002)

Type	*Classification of Problems*
A	*Direct Algorithm Usage*
A1	Problems where the arithmetic mean of a given dataset in table, graph, etc. format is asked
A2	Problems where the average and the number of data are given, and the total is asked
B	*Flexible Use of the Arithmetic Mean Algorithm*
B1	Problems where the arithmetic mean is given, and the missing elements of one data in the dataset are asked
B2	Problems where the arithmetic mean is given, and the missing elements of two or more data in the dataset are asked
B3	Problems where the arithmetic mean is given, and the elements of the dataset need to be constructed
B4	Problems involving the overall mean of multiple datasets
B5	Problems involving weighted averages

1.4. Strategies for Solving Arithmetic Mean Problems

"Center of balance," "Fair sharing," "Guess and check," and "Add-divide" are strategies applied in solving arithmetic mean problems (Cai, 2002; Marnich, 2008). The application of fair sharing and distribution center strategies is explained in solving the problem "In a science class, a student weighed an object ten times. The results of the weighing process are presented in the graph below. The student lost the results of the third and sixth weighing processes. If the average of the ten weights obtained from the weighing process is 3.2 grams, what could be the result of the third and sixth weighing processes?"

1.4.1. Fair Sharing (Redistribution)

The conceptualization of fair sharing can be explained using the idea of a signal or equal redistribution in a noisy process. For instance, the observed weights of an object on a balance scale can vary with each weighing. Changes in measurements can be thought of as a noisy process. The actual weight can be estimated with a signal that balances the arithmetic mean or variation (Konold & Pollatsek, 2002). Another scenario that reflects the idea of fair sharing is when each datum is represented by unit cubes, and the number of unit cubes in columns of equal length represents the arithmetic mean of the dataset (Van de Walle, Karp & Bay-Williams, 2013). When portions exceeding the arithmetic mean are distributed, as shown below (Figure 3), it can be determined that the results of the 3rd and 6th weighing processes are 3.2 grams and 3.1 grams, respectively.

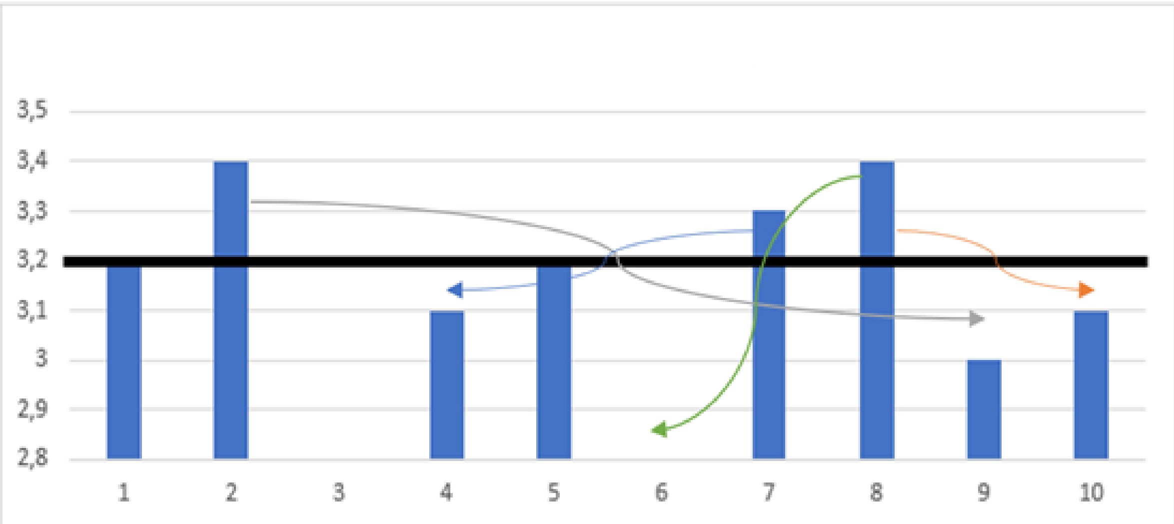


Figure 3. The Idea of Signal or Equal Redistribution in a Noisy Process

1.4.2. Center of Balance

Balance models help students understand the concept of center balance, that the arithmetic mean represents a value for a dataset, and how it is related to the characteristics and formula of the arithmetic mean (Ginat & Wolfson, 2002). They demonstrate to students that the relative frequencies of the scores are important in determining the average (Hardiman, Well, & Pollatsek, 1984, p. 794). Given that the arithmetic mean is 3.2, the seventh, eighth, ninth, and tenth measurement results balance each other. The values of the first and fifth measurements are equal to the arithmetic mean. The second measurement result is 0.2 greater than the arithmetic mean, and the fourth measurement result is 0.1 less than the arithmetic mean. Therefore, there is a difference of

+0.1. In this case, if we take the third measurement result as 3.2, we need to take the sixth measurement result as 3.1.

Problems that require creating a dataset (B2) or finding multiple elements (B1) instead of finding the arithmetic mean or a single element of a dataset are statistically more challenging. They limit the application of the arithmetic mean algorithm and direct students towards conceptual solutions. In such problems, the use of center balance and fair sharing strategies positively affects students' performance (Cai & Moyer, 1995; Hardiman et al., 1984; Mokros & Russell, 1995).

Guess and check and add-divide are the other two strategies used in solving arithmetic mean problems. These strategies are used to solve problems like "Ahmet is selling hats for families affected by an earthquake. The number of hats he sold in the first three weeks is given below. How many hats should Ahmet sell in the fourth week to have an average of 7 hats sold?" (Figure 4). The explanation of how these two strategies are applied to solve this type of problem is provided below (Cai, 2002).

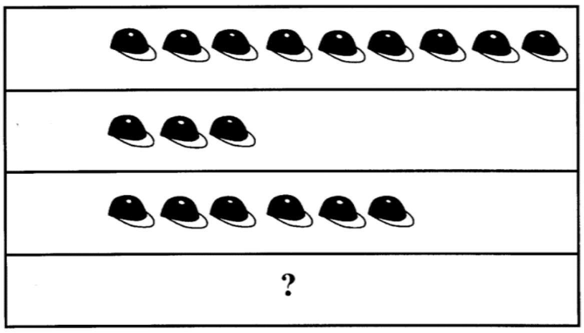


Figure 4. Visual Example Using the Add-Divide and Guess-Check Strategies

1.4.3. Add-divide

Students can perform arithmetic or algebraic solutions using the arithmetic mean formula. By using the arithmetic mean formula, a student solving it arithmetically can arrive at the answer of [(7x4-(9+3+6))=10]. A student solving it algebraically can achieve the same result by solving the equation (9+3+6+x)=7x4 (Cai, 2002).

1.4.4. Guess-Check

In the Guess-Check strategy, the student first selects a number for the fourth week, then checks whether the average of the sold hat numbers over four weeks is 7. If the average is not 7, the student continues to select a different number for the fourth week and checks again until the average becomes 7 (Cai, 2002).

In this study, the strategies used by high-achieving eighth-grade middle school students in solving problems related to the arithmetic mean algorithm and their performance on these problems were determined. In addition, one student from each school was interviewed and it was tried to determine how the concept of arithmetic mean was taught and what types of problems and solution strategies were used in the lessons. Then, the interview and written exam findings were compared and interpreted. The research questions to be answered in the scope of this study are presented below.

- What is the distribution of solution strategies used by middle school students while solving problems requiring flexible use of the arithmetic mean algorithm?
- What is the performance of middle school students on problems requiring flexible use of the arithmetic mean algorithm?

2. Method

The survey method was used to determine the strategies used by 8th grade students in solving arithmetic mean problems and their performance in arithmetic mean problems. In addition to having the power to investigate phenomena that have already occurred, the survey method can allow researchers to use non-probability sampling methods, including purposive sampling (Beins & McCarthy, 2011; DePoy & Gitlin, 2011). Purposive sampling can be chosen when researchers are interested in a specific type of person, such as students with math grades of 4 or 5 out of 5 (Beins & McCarthy, 2011). Even though there is no consensus among researchers concerning the minimum number of participants in such surveys, Cohen, Manion and Morrison (2013) suggested a minimum of 100 participants. The number of participants in this study is thought satisfactory.

2.1. Data Collection Tools

Five problems (see Appendix 1) related to the arithmetic mean concept was used, three of which were visual and two were verbal. The problems used in the study were designed to require flexible use of the arithmetic mean algorithm (Cai, Lo & Watanabe, 2002) and were developed based on previous studies (Cai, Moyer & Grochowski, 1999; Cai & Moyer, 1995; Konold ve Pollatsek, 2002; Uccellini, 1996). Since questions in verbal form and requiring direct use of algorithms are predominant in mathematics textbooks, questions in different forms and types were used in the study (Bütüner, 2020). The first three problems are in visual form, where in the first and second problems, the arithmetic mean is given and the missing element in the dataset is asked, and in the third problem, the arithmetic mean is given and the missing two elements in the dataset are asked. The last two problems are in verbal form, where in the fourth problem, the arithmetic mean is given and the missing element in the dataset is asked, and in the fifth problem, the arithmetic mean is given and the missing seven elements in the dataset are asked. The arithmetic mean test, consisting of five questions, was presented to expert researchers in the field and it was piloted to check the clarity of the items and the application time. The problems used in the study are provided in Appendix 1. After the written exam was administered, interviews were held with a total of 5 students, one student from each school. During the interviews, students were asked 4 questions (How was the concept of arithmetic mean taught by the teacher?, How were the problems related to arithmetic mean that you were asked?, In which form of representation are arithmetic mean problems asked in mathematics lessons?, How were arithmetic mean problems solved in the class?).

2.2. Data Analysis

In the first phase of the study, solutions provided by students to five problems were examined by two researchers. Within the scope of the study's objectives, the strategies used by students in problem-solving were identified. The strategies employed by students were coded as "add-divide", "center of balance," "fair sharing “and” guess-check". After identifying the strategies used by students in solving the five arithmetic mean problems, the performance of these students in the problems was analyzed. In the table regarding the students' performance, the number of correct and incorrect answers was given based on the strategy used. For example, for problem 1, the number of correct and incorrect answers of the students who used the add-divide algorithm was recorded in the table. If the student reached the correct result by using a strategy, the answer was coded as correct. If the student partially completed the solution by using a strategy but could not reach the correct result or reached the wrong result by making a calculation error, the answer was coded as incorrect. Students who did not solve the problem were not entered in the table, and the findings related to this were presented below the table. Two researchers came together to examine the student solutions and the interview data with the students and made codes.

2.3. Research Ethics

The author declared that the research has been approved by Yozgat Bozok Uludağ University Ethics Committee on 21 September 2022 with the protocol code 26/12.

3. Findings

The findings obtained from 103 eighth grade students with high achievement in mathematics are given below, respectively.

3.1. What is the distribution of solution strategies

To address the first research question, the number of strategies used by students in solving each of the five arithmetic mean problems was determined, and the results are presented in Table 3 along with frequency values. Upon analyzing students' solutions, it was found that there were no students who used more than two strategies. Therefore, in the table, the number of strategies is categorized as "one solution strategy" and "two solution strategies."

Table 3. Distribution of Solution Strategies

	Problem →	P1	P2	P3	P4	P5
Number of strategies used	*Strategy*	*f*	*f*	*f*	*f*	*f*
One solution strategy	Add-Divide	95	94	75	97	63
	Fair-Sharing	-	-	-	-	-
	Center of Balance	-	-	-	-	-
	Guess-Check	-	-	-	-	-
Two solution strategies	Add-Divide and Fair-Sharing	4	3	1	-	-
	Add-Divide and Center of Balance	-	-	-	1	1
	Add-Divide and Guess-Check	-	-	-	-	-
	Fair-Sharing and Center of Balance	-	-	-	-	-
No answer		4	6	27	5	39

When analyzing the strategies used by 103 students with mathematics grades of 4 or 5 in solving the six problems, it was found that nearly all students used the add-divide algorithm strategy. The number of students who used only the add-divide strategy for solving each problem is as follows: 95, 94, 75, 97, and 63, respectively. The number of students using multiple solution strategies is quite low. For the first, second, and third problems, 4, 3, and 1 student(s) respectively used both the add-divide and fair sharing (redistribution) strategies. For the fourth and fifth problems, 1 and 1 student respectively attempted to solve the problems using both the add-divide and center of balance strategies. Notably, a significant number of students couldn't answer these two problems. Specifically, 27 students couldn't answer the third problem, and 39 students couldn't answer the fifth problem. The number of students who couldn't answer the first, second, and fourth problems is 4, 6, and 5, respectively. In order to obtain more in-depth findings, interviews were conducted with one student from each school. The students were asked questions such as "How was the concept of arithmetic mean taught by the teacher?", "What were the problems related to arithmetic mean that you were asked?", "How were arithmetic mean problems solved in class?". During the interviews conducted with the students, all students mentioned that in the instruction of the arithmetic mean concept, the teacher introduced the add-divide algorithm and then solved several problems on the board using this rule. After that, the teacher posed similar problems to the students. The students emphasized that the add-divide strategy was used in solving arithmetic mean problems, and they noted that verbal problems in the form of A1, A2, and B1 types were used in relation to arithmetic mean. As a result, it was determined that the findings supported each other.

3.2. What is the performance of middle school students in solving problems

After identifying the strategies used by students in solving the five arithmetic mean problems, the performance of these students in the problems was analyzed. Table 4 displays the correct answer rates for students who used the (add-divide), (add-divide and fair sharing), and (add-divide and center of balance) strategies in the problems.

Table 4. Performance of Middle School Students

	P1		P2		P3		P4		P5	
Strategy	C	W	C	W	C	W	C	W	C	W
Add-Divide	85	10	85	9	51	24	90	7	37	26
Add-Divide and Fair Sharing	4-4	0-0	3-3	0-0	1-1	0-0	-	-	-	-
Add-Divide and Center of Balance	-	-	-	-	-	-	1-1	0-0	1-0	0-1
No Answer	4		6		27		5		39	

*C: correct; W: wrong

Only the numbers of correct answers for the students who used the add-divide algorithm are 85, 85, 51, 90, and 37, respectively. The numbers of incorrect answers are 10, 9, 24, 7, and 26, respectively. According to the obtained findings, the first, second, and fourth problems were correctly answered by the majority of the students. These problems involve giving the arithmetic mean and asking for an element from the dataset. Students can directly use the add-divide algorithm for this type of problem. Examples of the solutions of the students who answered the first and second problems correctly using the add-divide algorithm are presented in Figure 5.

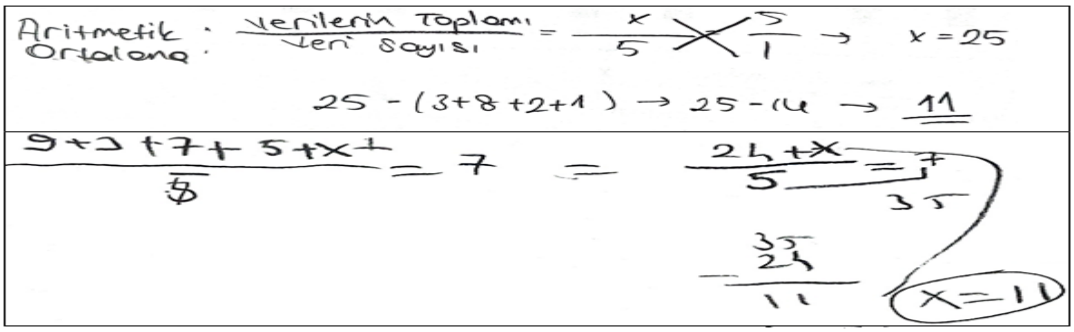


Figure 5. Examples of the Solutions of the Students who Answered the First and Second Problems
$[5x5 = 25, 25 - 14 = 11; 7x5 = 35, 35 - 24 = 11]$

Students who gave wrong answers to these problems (first, second and fourth problems) either made a calculation error or used the add-divide algorithm incorrectly (Figure 6). Looking at the above solution in Figure 6, the student used the add-divide algorithm correctly, but found the result of the 25-14 operation to be incorrect.

In the other solutions, we can say that the student made a mistake due to the incorrect use of the add-divide algorithm. While the student should have subtracted 24 from 35, he added the arithmetic mean and subtracted 31 from 35. In the third solution, while the number of data should be 5, the number of data is taken as 6 by including the arithmetic mean.

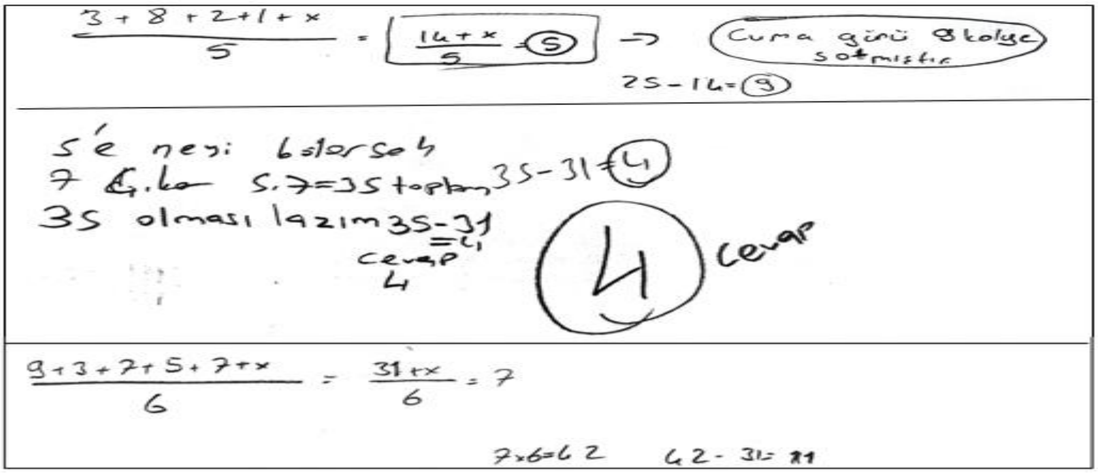


Figure 6. Calculation error or used the add-divide algorithm incorrectly

However, in the third and fifth problems, students were asked about multiple elements in the dataset given the arithmetic mean. It was observed that the performance of students dropped for these types of problems. The number of students who did not answer the third and fifth problem is 27 and 39, respectively. These students wrote on the written exam paper that they either did not understand the question or found it difficult. All the students who used both the add-divide and fair sharing strategies correctly answered the problems. For the first problem, the solution of a student using the add divide strategy and the fair share strategy is given in Figure 7.

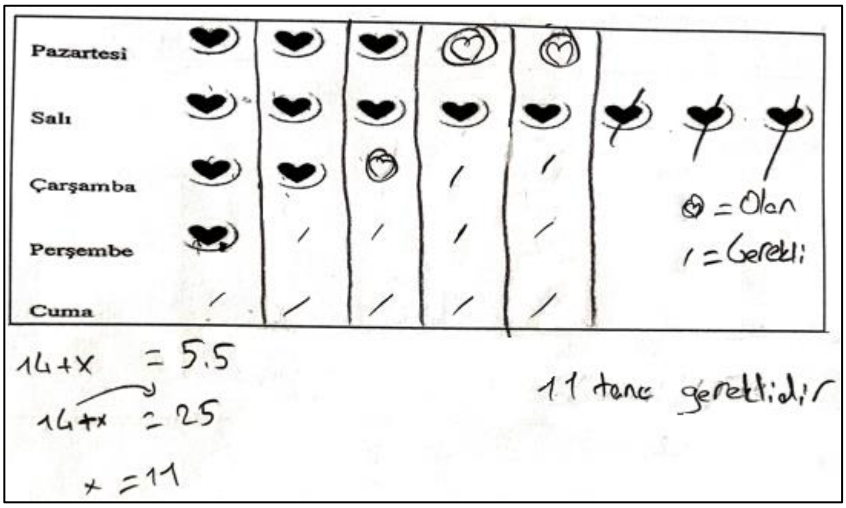


Figure 7. The Solution of a Student Using the Add Divide Strategy and the Fair Share Strategy

For each of the fourth and fifth problems, only one student used the add-divide and center of balance strategies. In the fourth problem, the student using both strategies gave the correct answer, in the fifth problem, the solution made with the add-divide strategy is correct, while the solution made with the center of balance strategy is wrong. The solution of the student who answered the fourth problem correctly using the add-divide and balance center strategies is given in Figure 8.

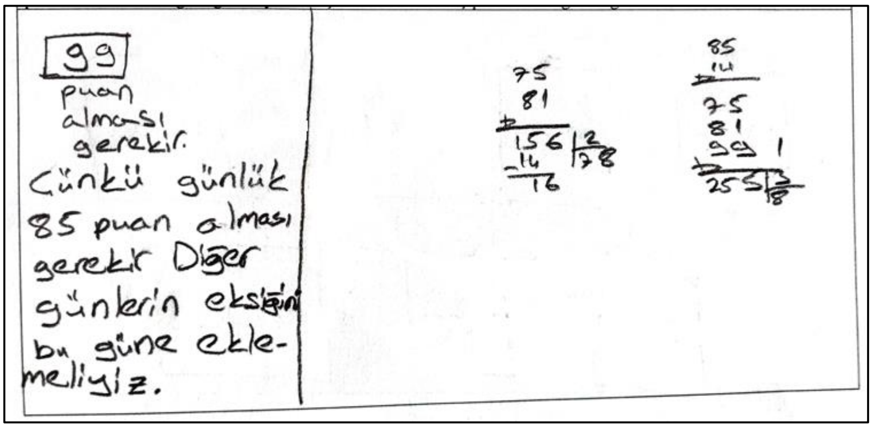


Figure 8. The Solution of a Student Using the Add Divide Strategy and the Center of Balance Strategy

In the third and fifth problems, the arithmetic mean was given and the students were asked what more than one element in the data set was. It has been determined that students' performances decrease in such problems and the number of students who cannot answer is higher than for the other problems. In addition, it was observed that some of the students who tried to answer the third and fifth problems with the add-divide strategy completed the solution by finding the sum of the elements of the data set, and some of them tried to create the desired elements by giving values after finding the sum of the elements of the data set. With this solution, there were students who correctly created the elements of the dataset that were not given, and there were also students who created the elements of the dataset incorrectly by making a processing error. Examples for both cases are given in Figure 7.

4. Discussion and Conclusion

Through the conducted analyses, it was determined that nearly all students applied to only the add-divide strategy in solving each of the given problems. The findings obtained from the interviews with the students confirm the above-mentioned findings. In the interviews with the students, all of the students stated that the concept of arithmetic mean is taught in a rule-oriented manner. In addition, they stated that their teachers only solve the arithmetic mean problems with the add-divide algorithm. Previously, Groth and Bergner (2006) sought to determine the knowledge of 46 pre-service teachers about mean, median and mode and found that most of them made insufficient explanations of the arithmetic mean. Similar studies indicate a consistent reliance on the add-divide algorithm (whether in algebraic or arithmetic form) by students when solving problems related to arithmetic mean (Cai, 1998, Cai, 2000, s.845, 848; Enisoğlu, 2014; Mokros & Russell, 1995, s.28-29; Uçar & Akdoğan, 2009). Cai (1998), in her study with 250 sixth grade students, found that the students knew the add-divide algorithm. More than half of the students (130 out of 250 or 52%) used one of three strategies (add-divide, guess-check, leveling). While the average formula is used most frequently among these strategies, the least used one is the leveling strategy. Specifically, 59% of the students (77 out of 130 students) used the average formula to solve the problem, 35% (45 out of 130 students) used the guess-check strategy and the remaining 6% (8 out of 130 students) used the leveling strategy. Cai (2000) also analyzed the responses of 311 sixth grade Chinese students and 232 sixth grade US students to two problems involving the arithmetic mean. Most of the students used the add-divide strategy in their problem solving. Akdoğan (2009) and Enisoğlu (2014) similarly found in their studies that students mostly use the add-divide algorithm when solving arithmetic mean problems.

In the present study, while most of the students correctly solved the first, second, and fourth problems, a decline in their performance was observed for the third and fifth problems. In her study Cai (1998) mentioned that only about half of the students used the add-divide algorithm correctly in solving problems. In another study, Cai reached similar results. Cai (2000) found that a significant portion of the students made mistakes due to the incorrect use of the add-divide algorithm. Student difficulties were not due to a lack of procedural knowledge of the averaging algorithm, but to a conceptual lack of understanding of the algorithm. Students who used more advanced representations were better problem solvers. Uçar and Akdoğan (2009) and Enisoğlu (2014) similarly found in their studies that the most common mistake students make is the misuse of the algorithm. In the current study, students' performance in problems was found to be better than previous studies. This may be because the participants were students with high mathematics achievement. However, the remarkable finding reached in the present study is that the number of students who did not answer the third and fifth problems or gave incorrect answers to them was high. The fact that the students used the add-divide algorithm more successfully may be due to the sample selection. However, although the students had high mathematics achievement, they could not show the same performance in the third and fifth problems in which more than one term was requested in a data set. Similarly, the interviewed students stated that A1, A2 and B2 type problems were included in the lessons. Therefore, The fundamental cause of students' difficulties with arithmetic mean lies in the fact that an algorithm-focused instruction was implemented without allowing them the opportunity to develop a conceptual understanding of the concept (Cai, 1998, 2000; Watson & Moritz, 2000).

In order to understand the statistical aspect of the arithmetic mean concept, it is important to first understand its mathematical aspect. Although the data for this study was collected from students with math scores of 4 or 5 out of 5, the high achievers in the study adhered to the add-divide algorithm for arithmetic mean problems. Therefore, being high achievers depended on how well students applied the rules and algorithms taught to them. Interviews with students indicated that one of the reasons for this result could be the teachers' teaching styles. Another reason could be how the concept of arithmetic mean is presented in mathematics textbooks, what types of problems and what solution strategies are included. Indeed, Bütüner (2020) stated in his study that the concept of arithmetic mean is taught in mathematics textbooks based on rules and that students are encouraged to use the add-divide algorithm in problem solving. It should be considered that the results obtained from the study may have various reasons other than teachers' teaching style and mathematics textbook.

Funding: This project coded SHD-2022-1061 was supported by Yozgat Bozok University Scientific Research Projects (BAP) Coordination Office. I would like to thank Yozgat Bozok University Scientific Research Projects (BAP) Coordination Office for their support. This study was produced from this project coded SHD-2022-1061.

Declaration of interest: The author declares no competing interest.

References

Beins, B. C., & McCarthy, M. A. (2011). *Research methods and statistics*. New Jersey (NJ): Pearson Higher Education.

Bowen, G. A. (2009). Document analysis as a qualitative research method. *Qualitative Research Journal, 9*(2), 27-40.

Bütüner, S. Ö. (2020). The learning opportunities presented by mathematics coursebooks used in middle schools in Turkey on the concept of arithmetic mean. *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 11*(1), 157-187. https://doi.org/10.16949/turkbilmat.627826

Bremigan, E. G. (2003). Developing a meaningful understanding of the mean. *Mathematics Teaching in the Middle School, 9*(1), 22-26.

Cai, J. (1995). Beyond the computational algorithm: Students' understanding of the arithmetic average concept. In L. Meria, & D. Carraher (Eds.), Proceeding of the 19th Psychology of Mathematics Education Conference (Vol. 3. pp. 144-151). Sao Paulo, Brazil: PME Program Committee.

Cai, J. (1998). Exploring students' conceptual understanding of the averaging algorithm. *School Science and Mathematics, 98*(2), 93-98.

Cai, J. (2000). Understanding and representing the arithmetic averaging algorithm: An analysis and comparison of US and Chinese students' responses. *International Journal of Mathematical Education in Science and Technology, 31*(6), 839-855.

Cai, J., Lo, J., & Watanabe, T. (2002). Intended treatmens of arithmetic average in U.S. and Asian School mathematics textbooks. *School Science and Mathematics, 102*(8), 391-404.

Cai, J., & Moyer, J.C. (1995). Middle school students' understanding of average: A problem solving approach. In D. T. Owens, M. K. Reed, & G. M. Millsaps (Eds.), *Proceedings of the Seventeenth Annual Meeting of the North American Chapter of the International Group of the Psychology of Mathematics Education* (Vol. 1, pp. 359-364). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Cai, J., Moyer, J.C., & Grochowski, N.J. (1999). Making the mean meaningful: An instructional study. *Research in Middle Level Education, 22* (4), 1-24.

Charalambous, C. Y., Delaney, S., Hsu, H.Y., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning, 12*(2), 117–151.

Chval, K. B., Chávez, Ó., Reys, B. J., & Tarr, J. (2009). Considerations and limitations related to conceptualizing and measuring textbook integrity. In J. T. Remillard, B. A.

Cohen, L., Manion, L., & Morrison, K. (2013). *Research methods in education*. 7th ed. London: Routledge.

DePoy E., & Gitlin, L. N. (2011). *Introduction to research: understanding and applying multiple strategies*. 4th ed. St. Louis (MO): Elsevier Health Sciences.

Enisoğlu, D. (2014). *Yedinci sınıf öğrencilerinin sütun grafiği gösteriminde verilen aritmetik ortalama, ortanca ve tepe değer kavramları ile ilgili problemleri çözerken kullandıkları olası çözüm stratejileri, yaptıkları hatalar ve yanlış yorumlamaları* (Yayımlanmamış yüksek lisans tezi). Orta Doğu Teknik Üniversitesi, Sosyal Bilimler Enstitüsü, Ankara.

Fan, L., & Kaeley, G. S. (2000). The influence of textbooks on teaching strategies: An empirical study. *Mid-Western Educational Researcher, 13*(4), 2–9.

Ginat, D., & Wolfson, M. (2002, July). On limited views of the mean as point of balance. Paper presented at the *26th Annual Conference of the International Group for the Psychology of Mathematics Education*, Norwich, England.

Guler, M., Gursoy, K., & Guven, B. (2016). Critical views of 8th grade students toward statistical data in newspaper articles: Analysis in light of statistical literacy, *Cogent Education, 3* (1), DOI: 10.1080/2331186X.2016.1268773.

Groth, R. E. & Bergner, J. A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning, 8*, 37-63.

Grouws, D. A., Tarr, J. E., Chavez, O., Sears, R., Soria, V. M., Taylan, R. D. (2013). Curriculum and implementation effects on high school students' mathematics learning from curricula representing subject-specific and integrated content organizations. *Journal for Research in Mathematics Education, 44* (2), 416-463

Hadar, L. L. (2017). Opportunities to learn: Mathematics textbooks and students' achievements. *Studies in Educational Evaluation, 55*, 153-166

Hardiman, P. T., Well, A. D., & Pollatsek, A. (1984). Usefulness of the balance model in understanding the mean. *Journal of Educational Psychology*, *76*(5), 792– 801.

Hirsch, C., Lappan, G., Reys, B., et al. (2005). Curriculum as a focus for improving school mathematics. *Mathematicians and Education Reform Forum Newsletter, 18*(1), 12–14.

Hong, D. S., & Choi, K. M. (2014). A comparison of Korean and American secondary school textbooks: The case of quadratic equations. *Educational Studies in Mathematics, 85*(2), 241–263.

Konold, C., & Higgins, T. (2003). Reasoning about data. In J. Kilpatrick, W. G. Martin, & D. E. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp.193–215). Reston, VA: National Council of Teachers of Mathematics (NCTM).

Konold, C., & Pollatsek, A. (2002). Data analysis as the search for signals in noisy processes. *Journal for Research in Mathematics Education, 33*, 259-289.

Leavy, A., & O'Loughlin, N. (2006). Preservice teachers understanding of the mean: Moving beyond the arithmetic average. *Journal of Mathematics Teacher Education, 9*, 53–90.

Lepik, M., & Grevholm, A. (2015). Viholainen Using textbooks in The mathematics classroom – the teachers' view. *Nordic Studies in Mathematics Education, 20* (3–4), 129-156

Lipka, J., & Adams, B. (2004). Culturally based math education as a way to improve Alaska native students' math performance Appalachian Collaborative Center for Learning.

Marnich, M. A. (2008). A knowledge structure for the arithmetic mean: Relationships between statistical conceptualizations and mathematical concepts (Unpublished doctoral dissertation). University of Pittsburgh, the USA.

Melis, E., Goguadze, G., Libbrecht, P., & Ullrich, C. (2009). Culturally adapted mathematics education with ActiveMath. *AI & Society, 24* (3), 251-265

Mokros, J., & Russell, S. J. (1995). Children's concepts of a average and representativeness. *Journal for Research in Mathematics Education*, *26*(1), 20-39.

Russell, S. & Mokros, J. (1996). Research into practice: What do children understand about average? *Teaching Children Mathematics, 2*(6), 360-364.

Sirnik, M., & Kmeti, S. (2010). Understanding of arithmetic mean. In C. Reading (Ed.), Data and context in statistics education: Towards an evidence-based society. Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia. Voorburg, The Netherlands: International Statistical Institute. www.stat.auckland.ac.nz/~iase/publications.php

Stein, M. K., Remillard, J., Smith, M. S. (2007). *How curriculum influences student learning*
F. Lester (Ed.), Second handbook of research on mathematics teaching and learning, Information Age Publishing, Charlotte, NC, pp. 319-370

Strauss, S., & Bichler, E. (1988). The development of children's concept of the arithmetic average. *Journal for Research in Mathematics Education*, *19*(1), 64-80.

Tarr, J. E., Chávez, Ó.., Reys, R. E., & Reys, B. J. (2006). From the written to the enacted curricula: The intermediary role of middle school mathematics teachers in shaping students' opportunity to learn. *School Science and Mathematics, 106* (4), 191-201

TIMSS (2011). *International Database*. Retrieved from https://timssandpirls.bc.edu/timss2011/internationaldatabase.

Uccellini, J. C. (1996). Teaching the mean meaningfully. *Mathematics Teaching in Middle School, 2*(3), 112-115.

Uçar, T. Z., & Akdoğan, N. E. (2009). İlköğretim 6–8. sınıf öğrencilerinin ortalama kavramına yüklediği anlamlar. *Elementary Education Online, 8*(2), 391–400.

Van Den Ham, A.-K., and Heinze, A. (2018). Does the textbook matter? Longitudinal effects of textbook choice on primary school students' achievement in mathematics. *Stud. Educ. Eval. 59*, 133–140. doi: 10.1016/j.stueduc.2018.07.005

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). Upper Saddle River, NJ: Pearson.

Watson, J. M., & Moritz, J. B. (2000). The longitudinal development of understanding of average. *Mathematical Thinking and Learning, 2*(1&2), 11–50.

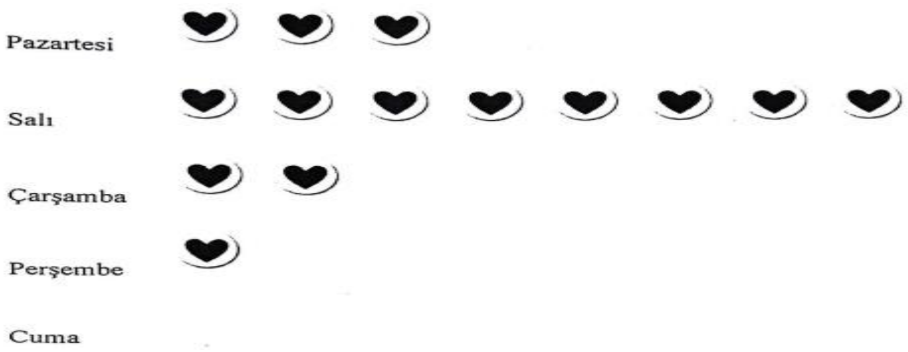
Watson, M. J. (2006). Statistical Literacy at School. London, LEA.

Weiss, I., Pasley, J., Smith, P., Banilower, E., & Heck, D. (2003). *Looking inside the classroom: A study of K-12 mathematics and science education in the US*. Chapel Hill. NC: Horizon Research.

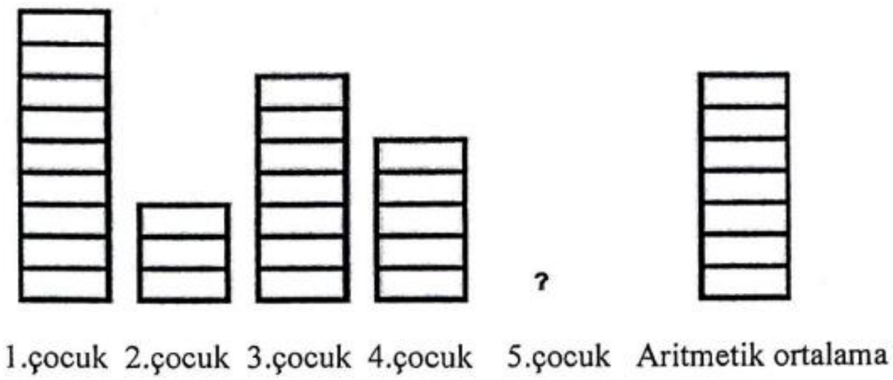
Zazkis, D. (2013). On students' conceptions of arithmetic average: The case of inference from a fixed total. *International Journal of Mathematical Education in Science and Technology, 44*(2), 204-213.

Appendix 1. The data collection instrument used in the study consists of five questions.

- At the charity fair for children with leukemia, the numbers of heart-shaped necklaces sold by Ayşe during the first four days of the week are given below. Given that the average number of heart-shaped necklaces sold by Ayşe during the weekdays is 5, find out how many necklaces she sold on Friday.

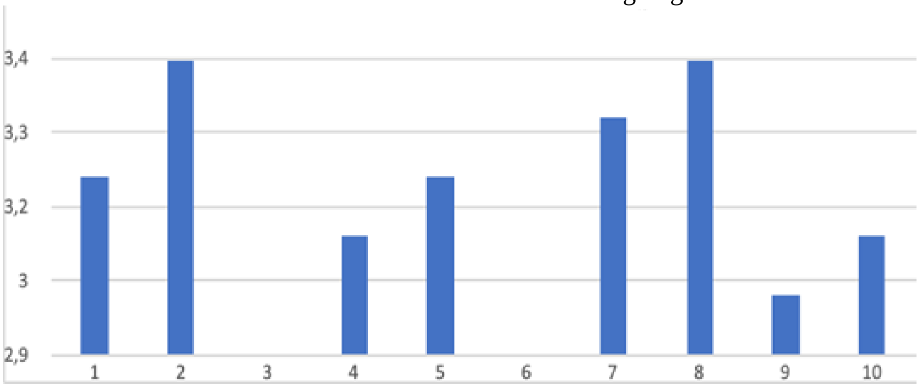


- The stacks of blocks in front of four children are given below. How many stacks of blocks should there be in front of the fifth child so that the average of the blocks is 7?



Appendix 1 continued

- In the science class, a student has weighed an object ten times. The results of the Weighing process are presented in the graph below. The student has lost the results of the third and sixth weighing. If the average of the ten weights obtained from the weighing process is 3.2 grams, what could be the results of the third and sixth weighing?



- Oya has scored 75 and 81 points on the first two exams of the math class. Since Oya wants her average score in the course to be 85, what score does Oya need to get on her third exam?

- You work as an employee at a supermarket, and you need to put price tags on 9 bags of potato chips. The average price of the chips is 12.50 TL, and you will not put 12.50 TL on any of the chip bags. Additionally, you will put a price tag of 13.50 TL on the first bag of chips and 12.00 TL on the second bag of chips. Determine the price tags for the remaining seven bags of chips.