Awareness Scale for the Nature of Transition from Arithmetic to Algebra: Validity and Reliability Study

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Abstract: This study was carried out to develop a valid and reliable measurement tool that can measure the awareness of mathematics teachers about the nature of the transition from arithmetic to algebra. The survey model, one of the quantitative research methods, was used in the study. Three hundred forty-five mathematics teachers participated in the study. Exploratory factor analysis was applied to the data obtained from 254 of these teachers, and confirmatory factor analysis was applied to the data collected from 91 of them. The results revealed that the "Awareness Scale for the Nature of Transition is valid and reliable.

Keywords: Arithmetic, Algebra, Algebraic thinking, Teaching material, Teacher training

1. Introduction

The first step of a person learning mathematics is to learn arithmetic. He then begins to learn advanced mathematics. Arithmetic takes its roots from numbers and deals with arithmetic numbers themselves (Radford, 2022; Xie, & Cai, 2022). According to Akkan (2009), arithmetic, the largest and most well-known branch of mathematics, includes four basic operations with numbers, all calculations based on these four operations, finding the unknown from the known, and the relationships between numbers. In other words, the first step of learning mathematics starts with arithmetic. Because to learn mathematics, you need to know arithmetic. Studies show that students form their ideas about algebra by structuring their experiences with arithmetic (Akkan, Akkan, & Güven, 2017; Kieran, 2022).

For this reason, students should have a good knowledge of arithmetic in order to be able to learn algebra well. This situation strongly connects algebra and arithmetic (Akkan, Baki, & Çakıroğlu, 2012; Kieran, 1992; Van Amerom, 2002). For students to pass from arithmetic to algebra, teachers must know the differences and relationships between arithmetic and algebra. In this context, the research was conducted to develop a measurement tool that could measure the awareness of mathematics teachers about the nature of the transition from arithmetic to algebra.

2. Theoretical Framework

2.1. Arithmetic

Numbers, relationships between numbers, four operations, and other calculations based on four operations (NCTM, 1989). According to Mason (2008), arithmetic is the process of finding the unknown from the known using four operations. As it can be understood from these definitions, arithmetic is the mathematical calculations and operations that arise from the needs of people in their daily life for which there is no theoretical basis for mathematics and are used for their needs (Özey, 2019). According to Akkan (2009), arithmetic, the largest and most well-known branch of mathematics, includes four basic operations with numbers, all calculations based on these four operations, finding the unknown from the known, and the relationships between numbers. In other words, the first step of learning mathematics starts with arithmetic. Because to learn mathematics, you need to know arithmetic.

Abstraction of the concept of arithmetic includes operations such as operations with numbers, comparison, and counting. It influenced the birth of algebra (Gürbüz & Toprak, 2014). In other words, it can be said that arithmetic takes its roots from number concepts, while algebra takes its roots from arithmetic. This situation shows that there is both a reciprocal and a strong relationship between arithmetic and algebra (Koçlar, 2019). Arithmetic and algebra cannot be separated from each other.

2.2. Algebra

Kieran (1992), algebra is a branch of mathematics, but it provides not only a representation of numbers and quantities with letter symbols but also calculations with symbols. According to Sutherland and Rojano (1993), algebra is a mathematical language used to explain ideas in mathematics or other disciplines. Sfard (1995)

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defined algebra as the general science of computation. For algebra, Usiskin (1997) states, “Algebra is a language of mathematics. The components that make up this language are unknowns, formulas, patterns, placeholders, and relations.” explained.

Vance (1998) defines algebra as generalized arithmetic or a language necessary to generalize arithmetic. Algebra is based on solid arithmetic and takes its roots from arithmetic (Akkan, Baki & Çakıroğlu, 2012). Students who learned arithmetic with numbers also entered algebra using symbols and letters (Palabiyik & İspir, 2011). Algebra, an essential branch of mathematics, emerged with the abstraction of arithmetic by comparison and operations with numbers (Akgün, 2006). This situation shows the necessity of knowing arithmetic in order to learn algebra.

2.3. Relationship between Arithmetic and Algebra

While the formation of arithmetic came from numbers, the formation of algebra was from arithmetic. While arithmetic takes roots from number concepts, algebra takes roots from arithmetic. This situation shows that there is both a reciprocal and a strong relationship between arithmetic and algebra (Koçlar, 2019). Arithmetic and algebra cannot be considered separately from each other. Students form the foundations of algebra by making generalizations with the given concepts (Carpenter & Levi, 2000). Algebra is not concerned with a few numbers as in arithmetic, but with all numbers and sets of numbers (Palabıyık & İspir, 2011). The transition process from arithmetic to algebra starts with the subject of patterns and focuses on discovering the relationships in the pattern and generalizing these relationships (Turan, 2013). In addition, Turan (2013) stated that expressing the patterns with symbols provides the formation of the basic concepts of algebra.

There are many studies in the literature that there is a significant relationship between arithmetic and algebra (Yıldızhan & Şengül, 2017). Some of those; stated that students structured their ideas about algebra based on their previous experiences with arithmetic (Kieran, 1992). According to Wagner (1983), students have difficulty understanding algebraic operations because of their inability to grasp the concept of numbers, which is the basic concept of arithmetic. Cooper, Boulton-Lewis, Athew, Willss, and Mutch (1997) stated that the deficiencies in understanding various structural and relational representations in arithmetic distract students from constructs that support algebraic thinking and cause them difficulties in algebra.

During the transition from arithmetic to algebra, students first make sense of the numerical relations of their situations (Çağdeşer, 2018). Then, students should discuss these relationships and express the results in letters (Çağdeşer, 2018). When we examine the connections between arithmetic and algebra, teachers need to give the connections between arithmetic and algebra to the students and emphasize that arithmetic and algebra are inseparable parts. At the same time, it is necessary to provide students with meaningful learning by establishing connections between the language of arithmetic and algebra (Akkan & Güven, 2017).

2.4. Differences between Arithmetic and Algebra

While there are similarities between arithmetic and algebra, there are also differences that distinguish arithmetic and algebra. For example, numbers and results are essential in arithmetic, while variables and processes are important in algebra. Malara-Navarra (2003), in his study on the differences between algebra and arithmetic, stated that while arithmetic deals with the result, algebra deals with the process. If we examine the results of Akkaya’s (2006) in his study to examine the differences and similarities between arithmetic and algebra, the table is as follows:

1. Arithmetic generally produces numerical solutions. Algebra generally generalizes by symbolizing problem-solving methods.
2. Algebra generalizes relations between numbers, while arithmetic generalizes exceptional number cases.
3. Tables are used to do calculations in arithmetic, while algebra is used as a problem-solving tool.
4. While arithmetic works with constant numbers, algebra deals with variables.
5. While arithmetic uses letters as abbreviations for objects, letters are used as variables or unknowns in algebra.
6. In arithmetic, it shows results with symbolic expressions, while in algebra it shows the process.
7. While the equal sign indicates the result in arithmetic, it indicates equivalence in algebra.
8. While arithmetic does the reasoning with the knowns, algebra does it with the unknowns.
9. Unknowns express the result in arithmetic, while in algebra, the unknowns express the starting point.
10. While arithmetic uses linear problems with one unknown, algebra uses problems that systems of equations can solve. It has been summarized.

Teachers should emphasize the relationship between algebra and arithmetic and that algebra and arithmetic are separate concepts independent of each other (Akkan & Güven, 2017). For this, teachers must plan the transition process from arithmetic to algebra well (Koçlar, 2019).

2.5. Literature Review

When the literature on mathematics education is examined, it is thought that there are problems related to the nature of algebra and what these problems can be. There are studies to eliminate these problems, but it is thought that the studies with teachers need to be sufficient to transfer the relationship between arithmetic and algebra to the students without creating misconceptions (Bozkaya, 2020). In this case, there are various studies conducted to increase students' algebra success in the studies examined. However, it is seen that there need to be more studies about how the transition from arithmetic to algebra should be and the methods to be used in the transition to algebra (Bozkaya, 2020). However, studies on the nature of the transition from arithmetic to algebra were generally conducted qualitatively, and quantitative studies were limited. Quantitative studies examining the perceptions about the nature of the transition from arithmetic to algebra are considered necessary in terms of generalizability. This research was carried out to develop a scale that can reveal the awareness of mathematics teachers about the nature of the transition from arithmetic to algebra. The study is unique in that it aims to develop a scale regarding the perceptions of the nature of the transition from arithmetic to algebra. The scale to be developed is essential for being the primary reference source for future quantitative research.

3. Method

3.1. Research Model

This study was carried out in the screening model, one of the quantitative research designs. Fraenkel, Wallen, and Hyun (2015) define survey studies as collecting data from large samples to reveal a general situation. In this study, this model was used as it was aimed to develop a valid and reliable "Awareness Scale for the Nature of Transition from Arithmetic to Algebra" for mathematics teachers.

3.2. Participants

The study group of research was formed in two stages. Two hundred fifty-four mathematics teachers working in public schools affiliated with the Ministry of National Education and volunteering to participate in the study participated in the first stage. The scale was prepared with the help of Google forms and delivered to the mathematics teachers who formed the research study group. 56% of the teachers participating in the study were female, and 44% were male. 82% of the participants are undergraduates, and 18% are graduates. The average age of the teachers is 28.62, and the average seniority is 4.96.

Ninety-one mathematics teachers working in public schools affiliated with the Ministry of National Education and volunteering to participate in the study participated in the second phase of the study. It was applied to the study group with the help of Google forms. Of the teachers participating in the study, 57 were female, and 34 were male. All of the participants are undergraduate graduates. The average age of the participating teachers is 31.16 and the average seniority is 7.25.

3.3. Preparation of the Trial Form of the Scale and the Process

In order to determine the awareness of mathematics teachers about the nature of the transition from arithmetic to algebra, the "Awareness Scale for the Nature of Transition from Arithmetic to Algebra" was developed by the researcher. In the development process of the scale, firstly, articles investigating the theoretical framework and the perceptions of the nature of the transition from arithmetic to algebra were examined. As a result of this examination, the researcher created an item pool consisting of 30 items. Then, the researchers and their advisors came together and held a meeting, and the first form of the scale was created with 21 items selected from the item pool. The remaining items were not included in the form because they needed to sufficiently explain the theoretical framework (do not reflect the transition process from arithmetic to algebra).

The form was first submitted to the expert opinion (one is an associate professor, the other a doctoral lecturer). In line with the opinion of the experts, it was stated that an item would not explain the transition from arithmetic to algebra, so the question was removed from the form. The form consisting of the remaining 20 questions was applied to five mathematics teachers, and a pilot study was conducted, and it was determined that all of the items were understandable. Then, the form was applied to 254 mathematics teachers.

3.4. Data Analysis

First, a reliability analysis was made on the data collected in the study, and the scale's internal consistency was examined. Cronbach Alpha, internal consistency coefficients were checked for the scale's reliability. The literature suggests using the Cronbach Alpha coefficient in testing the reliability of the scale (Bostic, & Sondergeld, 2015; Denisse & Sharon, 2017). SPSS 22 program was used in the analysis of the data.

In order to analyze the data, first of all, normality control was performed. In the analysis of normality, the Kolmogorov-Smirnov test was performed first. It is recommended to use the Kolmogorov-Smirnov test in cases where the sample size is 50 or more (Seçer, 2013). When the test results were examined, it was seen that $p>.05$.

This indicated that the data were normally distributed. The kurtosis values and skewness, histogram, Q-Q plot, and branch-leaf plots were examined. As a result of the analysis, it was determined that the mean and median were close to each other, and the kurtosis and skewness values were between ±1. In addition, when the Histogram, Q-Q Plot, and branch-leaf graphs are examined, it can be said that the awareness scale scores for the nature of the transition from arithmetic to algebra are normally-distributed.

In order to determine the factor structure of the scale, firstly, exploratory factor analysis (EFA) was performed with the data collected from a group of students; Then, the compatibility of the structure obtained with the data collected from a different group was tested with confirmatory factor analysis (CFA). Principal component analysis was performed to determine the factor structure of the scale. In order to determine the factor number of the scale, the scree plot was first examined. The item factor load value was examined for the validity of the items in the scale. Although different limits are taken for the item factor load value, it is stated that values of .40 and above will be appropriate (Hinkin, 1995; Howard, 2016). In this context, the minimum value for item factor load was taken as .40. Overlapping items were removed from the scale form. While naming the factors, the content of the items and the theoretical framework were considered. Pearson Product Moments correlation analysis was performed to determine the relationships between the factors. Since it was determined that the factors were related, oblique rotation (Direct Oblimin) was performed. In oblique rotation, the delta angle is taken as =0.0, and the kappa coefficient = 4.

4. Results

4.1. EFA of the awareness scale on the nature of the transition from arithmetic to algebra

Exploratory Factor Analysis, firstly, the suitability of the data for factor analysis was checked. For this, the KMO and sphericity assumptions were tested. KMO was .77, and the Barlett test X2 value was 330.932 ($p<.001$). The fact that the KMO was over .60 and the Barlett test was significant showed that the data were suitable for factor analysis (Seçer, 2015). Principal components analysis was applied to the original form of the Awareness Scale for the Nature of Transition from Arithmetic to Algebra. It was determined that the scale had a two-factor structure. Figure 1 shows the scree plot.

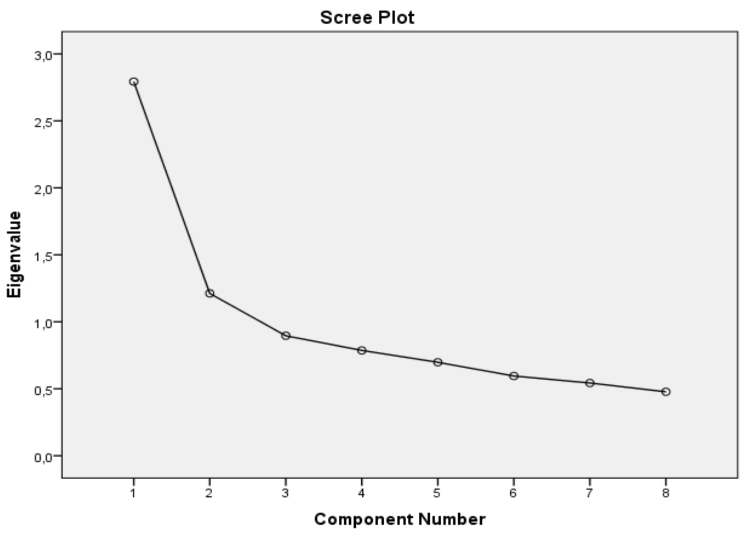

Figure 1. Slope slump plot

When Figure 1 is examined, it is seen that there are two dimensions with an eigenvalue vector size above one. The factors and eigenvalue vector magnitude are shown in Table 1. When Table 1 is examined, it has been determined that there are two dimensions with an eigenvalue vector size above one, and the variance value explained for both of these dimensions is above 5. Oblique rotation was made because the dimensions are related to each other. As a result of the rotation, a two-factor structure was obtained that explained 50.06% of the total variance. The items and factor loads in these factors are shown in Table 2.

Table 1. Table of Factors and Eigenvalue Vector Size

Total Variance Explained						
	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Component	total	% of Variance	cumulative %	total	% of Variance	cumulative %
1	2.793	34.915	34.915	2.793	34.915	34.915
2	1.212	15.149	50.064	1.212	15.149	50.064
3	.896	11.198	61.262			
4	.786	9.823	71.085			
5	.698	8.722	79.807			
6	.595	7.439	87.246			
7	.543	6.786	94.032			
8	.477	5.968	100.000			

Table 2. Table of Items and Factors Size.

	Component	
	1	2
I14	.82	
I10	.66	
I20	.59	
I7	.48	
I8	.47	
I2		−.84
I3		−.70
I4		−.57

When Table 2 is examined, items 14, 10, 20, 7, and 8 are in the 1st Dimension; It has been determined that Items 2, 3, and 4 are in the 2nd Dimension. It was determined that the first dimension explained 34.92% of the total variance, and the second dimension explained 15.15%. The item factor loading values in the scale. It was determined that it was over 47. The items showed that the first dimension focused on the differences between arithmetic and algebra, and the second dimension focused on the relationship between arithmetic and algebra. For this reason, the first of the dimensions is called the differences between arithmetic and algebra, and the second is called the relationship between arithmetic and algebra.

4.2. Confirmatory factor analysis (CFA) of the awareness scale on the nature of the transition from arithmetic to algebra

In order to examine whether the scale developed with EFA was verified or not, the 8-item form of the scale was applied to 91 mathematics teachers and Confirmatory factor analysis (CFA) was performed on the obtained data. The reliability of the collected data was calculated as 0.74. The item factor loading values obtained as a result of the CFA of the scale are presented in Figure 2.

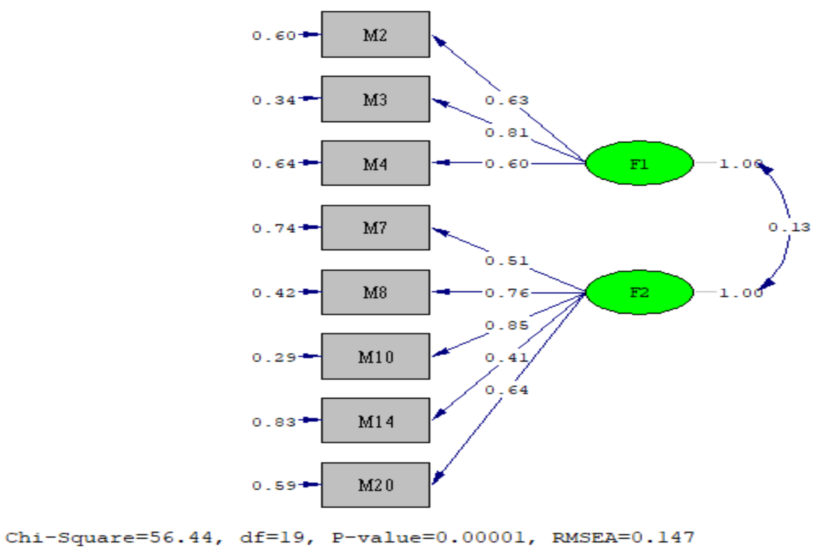

Figure 2. Confirmatory Factor Analysis of the Awareness Scale for the Nature of Transition from Arithmetic to Algebra

When Figure 2 is examined, item factor loading values are shown as a result of confirmatory factor analysis (CFA). According to the data obtained, when the fit index between the items is examined, it is seen that the chi-square value is (X^2= 56.44, N=91, df= 19, p= 0.00).

As a result of CFA, X^2/df= 56.44/19=2.97. In this framework, it is seen that the X^2/df value gives a perfect fit. The fit index values were RMSEA= .16, NFI= .79, NNFI= .78, CFI= .85, IFI= .85, RFI= .70, RMR= .078, GFI=.85 and AGFI=.72. These fit index values show that the model fits well. As a result, it can be concluded that the structure established by the researcher is confirmed.

As a result of the reliability analysis, it was found that the data were reliable (α=.72). According to the confirmatory factor analysis of the awareness scale about the nature of the transition from arithmetic to algebra, a scale consisting of eight items and two factors was obtained.

5. Conclusion and Discussion

The findings of this research, which aims to develop a valid and reliable measurement tool that can measure the awareness of mathematics teachers about the nature of the transition from arithmetic to algebra, support the literature. The results obtained in the research showed that the awareness scale for the transition from arithmetic to algebra consists of two dimensions: the differences between arithmetic and algebra and the relationship between arithmetic and algebra. In this respect, the scale supports the literature.

As a result of the study, the "Awareness Scale for the Nature of Transition from Arithmetic to Algebra" was developed, consisting of eight items and two dimensions, whose validity and reliability were ensured. Research on the relationship between arithmetic and algebra has generally focused on the relationship between arithmetic and algebra and the differences between arithmetic and algebra (Akkan et al., 2011; Akkan et al., 2017; Kieran, 1992; Van Amerom, 2002). The scale developed in this study is consistent with the literature.

When the items related to the relationship between arithmetic and algebra were examined, it was determined that the first item was related to seeing arithmetic as a subset of algebra. Akkan et al. (2011) stated that algebra is more comprehensive than arithmetic and is based on arithmetic. Studies have stated that students structure their ideas about algebra based on their previous experiences with arithmetic (Kieran, 1992). In this context, the item regarding arithmetic as a subset of algebra is an item for the relationship between arithmetic and algebra. The two items on the relationship between arithmetic and algebra aim to see algebra as a generalized version of arithmetic. Vance (1998) defined algebra as generalized arithmetic or a language necessary to generalize arithmetic. Carpenter and Levi (2000) emphasized that students construct algebra by generalizing with the given concepts. In this context, the fact that the related items are in the dimension of arithmetic and algebra relationships supports the literature.

The second dimension of the developed scale focused on the differences between arithmetic and algebra. The first item is "I think that making connections between operations in arithmetic helps to understand algebra ". There are important findings in the literature that establishing a relationship between students' arithmetic operations facilitates the transition to algebra (Akkan, 2009; Akkan, et al., 2017). In the studies on the differences between arithmetic and algebra, it has been stated that the unknown in arithmetic is the result and the process in algebra (Usiskin, 1997; Van Amerom, 2002). In the scale of this study, it can be said that the item " I think that unknowns are processes in algebra and results in arithmetic" overlaps with the literature. The literature states that arithmetic is related to counting and includes concrete operations; He emphasized that algebra is based on arithmetic and includes abstract operations (Akkan et al., 2017; Kieran, 1992; Öztürk, 2021). It can be said that the item "I think that arithmetic includes concrete operations while algebra includes abstract operations" in this study overlaps with the literature. Studies on the differences between arithmetic and algebra have revealed that letters are seen as placeholders and as variables in algebra (Akkan et al., 2011; Kieran, 1992; Van Amerom, 2002). It can be said that the article "letters are placeholders in arithmetic while letters are variable in algebra" in this study supports the literature. Research on the differences between arithmetic and algebra; emphasized that algebra requires dealing with unknowns (Akkan et al., 2011; Kieran, 1992; Van Amerom, 2002). It can be said that the code "algebraic deals with unknowns while operating with arithmetic numbers" in this study supports the literature.

The scale developed in the study explains approximately 50% of the total variance. Beavers et al. (2013) stated that the explained variance rate should be between 75-90%, but a value of 50% or more is acceptable. In this context, the explained variance rate is appropriate according to the literature. Erkuş (2012), on the other hand, emphasized that the specified standard values are not evidence-based and that the explained variance value may vary depending on the number of dimensions from its scale. He said a variance ratio explained for each scale dimension above 10% would be a sufficient criterion. Each dimension of the scale developed in this study has an explained variance rate of over 10%. In this respect, it can be said that the explained variance rate of the scale is sufficient.

The study determined that the lowest item factor load value was .47. Although different limits are taken for the item factor load value in the current literature, it is stated that values of .40 and above would be appropriate (Hinkin, 1995; Howard, 2016). In this context, the item factor load value calculated in the research is appropriate. The current literature shows that when the item factor load differences in more than one factor in the scale are over .10, there will be overlapping items (Büyüköztürk, 2007; Erkuş, 2012; Howard, 2016). For this reason, overlapping items were excluded from the scale.

When the findings for confirming the scale structure were examined, it was determined that all scale items were appropriate, and the structure was confirmed. The reliability analysis carried out determined the internal consistency coefficient of the scale as .72. Bostic and Sondergeld (2015) emphasized that a reliability value of .70 and above indicates sufficient, a reliability value of .80 and above indicates good, and a reliability value of .90 and above indicates excellent fit. The internal consistency coefficient reached in this research is sufficient in this context.

6. Suggestions

The literature shows studies on awareness of the nature of the transition from arithmetic to algebra. However, the number of quantitative studies in which the research is carried out qualitatively could be much higher. The "Awareness Scale for the Nature of Transition from Arithmetic to Algebra" was developed in the study. It is expected to be an incentive for quantitative research in the literature. In this context, future researchers can plan intervention studies to raise awareness of the nature of the transition from arithmetic to algebra. In addition, large-scale survey studies and relational studies can be conducted to examine the awareness of mathematics teachers about the nature of the transition from arithmetic to algebra in terms of various variables. In addition, studies can be carried out to adapt the scale to different languages.

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References

Akgün, L. (2006). Cebir ve değişken kavramı üzerine. *Journal of Qafqaz University*, *17*(1), 25-29.

Akkan, Y., Akkan, P., & Güven, B. (2017). Aritmetik ve cebir kavramları ile ilgili farkındalık. *Bayburt Eğitim Fakültesi Dergisi, 12*(20), 527-558.

Akkan, Y., Baki, A. & Çakıroğlu, Ü. (2012). 5-8. sınıf öğrencilerinin aritmetikten cebire geçiş süreçlerinin problem çözme bağlamında incelenmesi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *43,* 1-13.

Akkan, Y. (2009). *İlköğretim öğrencilerinin aritmetikten cebire geçiş süreçlerinin incelenmesi* (Doktora tezi- Tez No.259860). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir.

Akkaya, R., & Durmuş, S. (2006). İlköğretim 6-8. sınıf öğrencilerinin cebir öğrenme alanındaki kavram yanılgıları. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *31*, 1-12.

Bostic, J. D., & Sondergeld, T. A. (2015). Measuring sixth-grade students' problem solving: validating an ınstrument addressing the mathematics common core. *School Science and Mathematics, 115*(6), 281-291.

Bozkaya, C. (2020). *Aritmetik işlemlerden cebire geçişte öğretmen yaklaşımlarının incelenmesi* (Yüksek lisans tezi- Tez No.643623). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir.

Carpenter, T.P., & Levi, L. (2000). Developing conceptions of algebraic reasoning in the primary grades. Retrieved March 11, 2008, from www.wcer.wisc.edu/ncisla /publications/ index.html

Cooper, T. J., Boulton-Lewis, G., Athew, B., Willss, L. & Mutch, S. (1997). The transition arithmetic to algebra: Initial understandings of equals, operations and variable. *International Group for the Psychology of Mathematics Education*, *21*(2), 89-96.

Çağdaşer, B.T., (2008). *Cebir öğrenme alanının yapılandırmacı yaklaşımla öğretiminin 6. sınıf öğrencilerinin cebirsel düşünme düzeyleri üzerindeki etkisi (*Yüksek lisans tezi- Tez No.231211). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir.

Denisse, R. T. & Sharon, L. S. (2017) Examining content validity of tests using teachers' reported opportunity to learn, *Investigations in Mathematics Learning*, *9*(3), 148-155. https://doi.org/10.1080/19477503.2017.1310572

Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2015). *How to design and evaluate research in education*. New York: McGraw-hill.

Gürbüz, R. & Toprak, Z. (2014). Aritmetikten cebire geçişi sağlayacak etkinliklerin tasarlanması, uygulanması ve değerlendirilmesi. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi, 8*(1), 178-203.

Hinkin, T. R. (1995). A review of scale development practices in the study of organizations. *Journal of management*, *21*(5), 967-988. https://doi.org/10.1177/014920639502100509

Howard, M. C. (2016). A review of exploratory factor analysis decisions and overview of current practices: What we are doing and how can we improve? *International Journal of Human-Computer Interaction*, *32*(1), 51-62. https://doi.org/10.1080/10447318.2015.1087664

Kieran, C. (1992). The learning and teaching of school algebra. D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 334-370). Macmillan Publishing Co, Inc.

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: background, overarching dimensions, and new directions. *ZDM Mathematics Education, 54*(6), 1131–1150. https://doi.org/ 10.1007/s11858-022-01435-6

Koçlar, N. (2019). *Yaratıcı drama yöntemiyle cebirsel ifadelerin öğretimi* (Yüksek lisans tezi- Tez No.564228). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir.

Malara-Navarra, A. (2003), Dialectics between theory and practice: theoretical issues and aspects of practice from an early algebra project, in N.A. Pateman, B.J. Dougherty e J.T. Zilliox (a cura di), Proceedings of PME 27, Honululu (USA), vol.1, pp. 33-48.

Mason, J. (2008). Making Use of Children's Powers to Produce Algebraic Thinking. In J. Kaput, D. Cerraher, & M. Blanton (Eds), *Algebra in the early grades* (pp. 57-94). Mahwah, New Jersey: Lawrence Erlbaum.

NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

Özey, K. (2019). *Cebir Öğrenme Alanında Yapılan Lisansüstü Tezlerin İncelenmesi: 2010-2018 Yılları Arası Türkiye Örneği* (Yüksek lisans tezi- Tez No.601771). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir.

Öztürk, M. (2021). An embedded mixed method study on teaching algebraic expressions using metacognition-based training. *Thinking Skills and Creativity*, *39*, 1–15.

Palabıyık, U. & Akkuş İspir, O. (2011). Örüntü temelli cebir öğretiminin öğrencilerin cebirsel düşünme becerileri ve matematiğe karşı tutumlarına etkisi, *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 30*(30), 111-123.

Radford, L. (2022). Introducing equations in early algebra. *ZDM Mathematics Education, 54*(6), 1151–1167. https://doi.org/10.1007/s11858-022-01422-x

Sfard, A. (1995). The development of algebra: Confronting historical and psychological perspectives. *Journal of Mathematical Behavior*, *14*(1), 15-39.

Sutherland, R., & Rojano, T. (1993). A spreadsheet approach to solving algebra problems. *The Journal of Mathematical Behavior*, *12*(4), 353–383.

Turan, P. (2013). *Değişken kavramının öğretimi sürecinde elektronik tablo kullanımı: Bir öğretim deneyi* (Yüksek lisans tezi- Tez No.342588). Yükseköğretim Kurulu Ulusal Tez Merkezi'nden edinilmiştir.

Usiskin, Z. (1997). Doing algebra in grades K-4. *Teaching children mathematics*, *3*(6), 346-356.

Van Amerom, B. A. (2003). Focusing on informal strategies when linking arithmetic to early algebra. *Educational Studies in Mathematics*, *54*, 63-75.

Vance, J. H. (1998). Number operations from an algebraic perspective. *Teaching children mathematics*, *4*(5), 282-285.

Wagner, S. (1983). What are these things called variables? *Mathematics Teacher*, *76*(7), 474-479.

Xie, S., Cai, J. (2022). Fifth graders' learning to solve equations: the impact of early arithmetic strategies. *ZDM Mathematics Education*, *54*(6), 1169–1179. https://doi.org/10.1007/ s11858-022-01417-8

Yıldızhan, B., & Şengül, S. (2017). 6. sınıf öğrencilerinin harflerin anlamına yönelik kavram yanılgılarının aritmetikten cebire geçiş süreci bağlamında incelenmesi ve öğrencilerin matematik tutumları ve öz yeterlikleri ile karşılaştırılması. *The Journal of International Lingual Social and Educational Sciences, 3*(2), 249-268.

Appendix 1. Awareness Scale for the Nature of Transition from Arithmetic to Algebra

Dear Teachers,

The scale below was developed to determine the awareness of mathematics teachers about the nature of the transition from arithmetic to algebra. Please read the items carefully and choose the option that best suits you. It will take you about five minutes to complete the scale. Only voluntary participants should fill in the scale. If you do not volunteer to participate, please notify the practitioner.

Thank you for your participation.

Previous Item Number	Current Item Number	Items	Strongly disagree	Midly disagree	Equally agree and disagree	Midly agree	Strongly agree
2	**1**	I see arithmetic as a subset of algebra.					
3	**2**	I see algebra as a generalization of arithmetic.					
4	**3**	I see algebra as a more advanced form of arithmetic.					
7	**4**	I think that making connections between operations in arithmetic helps to understand algebra.					
8	**5**	I think the unknowns are the result in arithmetic and the process in algebra.					
10	**6**	I think that arithmetic includes concrete operations while algebra includes abstract operations.					
14	**7**	In arithmetic, letters are placeholders, while in algebra, letters are variables.					
20	**8**	While arithmetic deals with numbers, algebra deals with unknowns.					

Note. The original version of the scale is Turkish. Please contact the corresponding author for the original version of the scale.