Development and Validation of a Questionnaire to Explore Teachers' Knowledge about the Nature of Mathematics the Teaching and Learning of Mathematics

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Article History: Received: 1 June 2023; Accepted: 18 August 2023; Published online: 31 August 2023

Abstract: Investigating teachers' knowledge about the nature of mathematics and their viewpoints about the teaching and learning of mathematics is important as it has a great influence on what they do in classrooms. This paper describes the two phases of the development and validation process of an instrument named as Mathematics Teacher Survey Questionnaire (MTSQ) developed to determine Pakistani teachers' knowledge about the nature of mathematics, its influence on their viewpoints about the teaching and learning of mathematics. Phase one includes item writing, item analysis and item administration whereas phase two includes the tool validation process using Cronbach's alpha and content validity to ensure the developed tool is reliable and valid. An acceptable score of reliability indicates that MTSQ is an appropriate tool to investigate teachers' knowledge about the nature of mathematics and how the teachers' viewpoints are shaped. Exploratory Factor Analysis (EFA) was conducted on 37 items scale by administered to 200 teachers from public and private schools in Karachi, Pakistan.

Keywords: Nature of mathematics, Teaching and learning of mathematics, Tool development, Pakistan

1. Introduction

Educational systems all over the world including Pakistan are under constant pressure to adapt to the changing needs of society and impart quality education to learners. Hence, within mathematics education in the Pakistani context and elsewhere, a variety of initiatives is being taken to promote quality teaching and learning in school contexts. In Pakistan, initiatives such as national curriculum reforms, textbook reviews, material development, Pre-STEP[1] and STEP[2], for supporting in-service teachers to improve the knowledge and skills required to teach mathematics are undertaken. There is a wide consensus that the complexity of education is increasing the expectation from teachers to teach effectively. However, mathematics teaching and learning is not yet considered to have reached a desirable state, particularly in Pakistan even though teachers participate in numerous school-based training programs. One of the possible reasons for the minimal improvement in mathematics teaching and learning could be due to a mismatch between teachers' perceived views about mathematics, mathematics teaching and learning and what teacher education programmes offer and promote. Thompson (1984) asserts that:

> Any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by the teachers and how these are related to their instructional practice. Failure to recognize the role that the teachers' conceptions might play in shaping their behaviour is likely to result in misguided efforts to improve the quality of mathematics instruction in the schools. (p. 106)

Thompson argues that teachers' teaching practice is influenced to a greater extent by their perceptions and beliefs about the subject they teach and their views about learning and teaching. Similarly, Lerman (1990) asserts that unless teachers' knowledge about mathematics, mathematics teaching and learning are examined, "little will be achieved in terms of development and change in the mathematics classroom" (p. 54). Hence, understanding teachers' viewpoints is important, particularly when educational reforms are launched, as research evidence shows that teachers' personal theories about mathematics have a great influence on what they do in classrooms which further influences students' attitudes towards mathematics (Çelik, 2021; Barkatsas, 2008; Gates, 2006; Halai, 2001; Lloyd, 2002; & Schoenfeld, 1992).

Pakistan is a predominantly Muslim country where religion plays a significant role in shaping social and cultural practices. In this context teachers' beliefs and conceptions about the nature of knowledge are also influenced by the socio-religious context. As there has not been any research-based tool that can offer insight into teachers' perspectives, developing the tool was an important contribution. Hence, developing MTSQ serves the purpose of constructing a scientifically developed tool to investigate teachers' knowledge about the nature of mathematics and how the teachers' viewpoints are shaped in the socio-religious context of Pakistan.

[1] Pre-Service Teachers Education Program (Pre-STEP) - http://www.usaid.gov/pk/sectors/education/pre-step.html
[2] Strengthening Teacher Education in Pakistan (STEP) http://www.usaid.gov/pk/sectors/education/step.html

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Citation Information: Amirali, M. (2023). Development and validation of a questionnaire to explore teachers' knowledge about the nature of mathematics the teaching and learning of mathematics. *Turkish Journal of Mathematics Education*, *4*(2), 59-66.

2. Literature Review

There are various perspectives through which philosophers have discussed the origin of knowledge in general and the nature of mathematical knowledge in particular. In general, the two dominant epistemological perspectives of mathematical philosophies are 'absolutism' and 'fallibilism' (Ernest, 1991; Lerman, 1990; Jaworski, 1996). Dossey (1992) and Thompson (1992) based on a thorough synthesis of the literature on conceptions of the nature of mathematics assert that the teachers' viewpoints have a major influence on the implementation of the school curriculum, instruction and learning promoted in the school context. Ernest (1985), Dossey (1992) and Kulikowich and DeFranco (2003) draw on the discussion of the nature of mathematics as far back as the fourth century BC, with Plato and Aristotle as the two main contributors to the philosophical argument about the nature of mathematics. Plato took the position that "the objects of mathematics had an existence of their own, beyond the mind, in the external world" (Dossey, 1992, p. 40). Hence, Plato held a view that there is a world of ideas and a world of things, and that mathematics is the essence of the ideal world. Aristotle's view of mathematics was based on "experienced reality, where knowledge is obtained from experimentation, observation and abstraction" (Dossey, 1992, p. 40). This view supports the conception that one constructs the relations inherent in a given mathematical situation both through senses and abstraction. Thus, the construction of a mathematical idea comes through idealizations performed by the mathematician as a result of experiences with mathematical ideas and objects. Hence, it is considered that mathematical knowledge can only make sense when humans experience it. Both viewpoints indicate an absolutist view of mathematics

The fallibilist view emerged when the universality, absoluteness and perfectibility of mathematical knowledge was questioned (Ernest, 1999). Ernest (1991) further elaborates that fallibilists claim 'the impossibility of complete certainty' in mathematics, and he agrees that in many cases mathematical knowledge has an empirical basis and considers the mathematical activity to be a human activity. This means that mathematical knowledge is accepted based on both the empirical as well as theoretical i.e. on previously formed concepts. Ernest (1991) quotes an example to explain the theoretical ground of mathematical knowledge acquisition "I know 999,999 + 1 = 1,000,000 not through having observed its truth in the world, but through my theoretical knowledge of number and numeration" (p. 34). Thus, mathematical knowledge has an empirical origin (based on observation of the physical world) or is based on theoretical grounds.

Ernest (1991) further simplified the application of philosophical stance - instrumentalist, platonist or problem-solving view of mathematics in the teaching and learning process. According to Ernest, one can have an Instrumentalists view mathematics similar to the toolbox, as an accumulation of facts, rules and skills to be used in the pursuance of some external end. This means that mathematics is considered as a set of tools and knowing mathematics is to know what tools you have and how to use them and when. Thus, mathematics is seen as a set of unrelated but utilitarian rules and facts. Mathematics teachers holding instrumental views will consider themselves as masters having and imparting mathematical knowledge. Platonists view mathematics as a "static, but unified body of knowledge, a crystalline realm of interconnecting structures and truth, bound together by filaments of logic and meaning. Thus, mathematics is a monolithic, a static immutable product" (p. 132). This means that platonists focus more on the holistic approach, knowing how various tools work together and what makes them work. Mathematics teachers holding platonist views would try to find linkages among the mathematical concepts rather than considering them as unrelated rules and facts. A problem-solving view of mathematics encompasses mathematics as a dynamic, continually expanding field of human creation and everchanging field with inventions generating patterns and then distilled into knowledge. Mathematics being a human endeavour is not considered to be discovered, rather it is created. Therefore, mathematics is considered as a process of inquiry where knowing mathematics is equated with doing mathematics.

Kukari (2004) and Mansour (2008) add another dimension and highlighted the importance of religious and cultural views on individuals' practice. Kukari asserts that teachers' views about teaching as the transmission of prescribed knowledge from the teacher to the learners and learning as the absorption and the memorization of prescribed knowledge resonate with their cultural and religious practices. Kukari further elaborated that it is a cultural norm and religious practice, for the three teachers in her study, that the learners have to sit quietly and listen attentively to those who are responsible for transmitting knowledge without questioning to show respect for adults. Then, the learners memorize and practice what is being taught. Pakistan being socio-religious context this aspect was also considered important while constructing the tool. Thus, reviewing the available tools revealed that in Pakistani context where the socio-cultural beliefs and practices are dominant, there was a need to capture nuances emerging from the contextual realities which are often not readily visible or identifiable. Hence, in developing the survey questionnaire contextual realities with reference to teachers' preconceived beliefs and understanding about mathematics and teaching and learning of mathematics shaped by their social, cultural and religious experiences were taken into consideration.

3. Methodology

The MTSQ tool construction process was carried out in two phases. Phase one consists of item writing, item analysis and item administration whereas phase two included tool validation process using Cronbach's alpha and content validity.

3.1. Phase One

3.1.1. Item Writing

The survey questionnaire was developed on the basis of my extensive experience of teaching mathematics in a Pakistani context, the literature review in the area of the study and available survey questionnaires. The development of the questionnaire started with the process of item pooling under the themes i.e., perception of the nature of mathematics, teaching and learning of mathematics and teaching practices, this being the focus of the study. The research-based tools were referred for generating items for the MTSQ with prior permission (International Test Commission, 2017). The survey questionnaires referred to while developing survey items for the study were 'Mathematics Belief Scale' (Margaret, 2001); 'Attitudes and Beliefs about the Nature of and the Teaching of Mathematics and Science' (McGinnis, Kramer, & Watanabe; 1998); 'Attitude towards Mathematics Inventory' (Curtis, 2006); 'Perception of mathematics and mathematics education' (Lerman, 1990); and "Conception and attitude towards mathematics' (Amirali, 2007). While developing MTSQ items from the research-based tools most of the items were reworded or rephrased to make it simple for teachers to read and interpret. For instance, items from McGinnis, Kramer, and Watanabe (1998) were rephrased such as 'Mathematics consists of unrelated topics (e.g., algebra, arithmetic, calculus and geometry) to 'Mathematics comprises only formulae, symbols and rules' and 'Using technologies in mathematics lessons will improve students' understanding of mathematics' to 'Use of a calculator hinders students' understanding of mathematics'. Whereas, one of the items from Lerman (1990) 'Mathematical truths are not susceptible to revolutionary change in the way that scientific truths are, e.g.' relativity' was reworded as 'Current mathematical knowledge will remain the same in the future'.

Overall, the item developed were thirty-seven.

3.1.2. Item Analysis

To establish the content validity of the items constructed, the item pool was given to seven experts with proficiency in scale development and seven with the construct of the nature of mathematics, teaching and learning of mathematics. The experts were asked to rate items on a 5-point scale ranging from 1-least relevant to 5-Most relevant based on their clarity, accuracy, culture fairness, comprehension, and appropriateness. Descriptive feedback and comments were also obtained. The experts' evaluation recommended the elimination and modification of items that were repetitive, lengthy, and conceptually inconsistent items.

3.1.3. Survey Administration – Pilot Study

To assess the suitability of instructions and whether the respondents encountered any difficulty, the survey questionnaire was administered to mathematics teachers. The sample included thirty teachers selected based on convenient sampling from both public and private schools. The pilot study revealed that the teachers did not encounter any major problems regarding comprehension of the survey questionnaire.

3.1.4. Survey Administration – Final Study

In the final study, 16 public schools and 21 private schools were selected through stratified random sampling from the strata i.e., public and private schools. Overall, 200 secondary school mathematics teachers gave consent to participate in the study. The survey was self-administered. In the public schools, school heads allowed the research to interact with the teachers and collect the filled survey questionnaire whereas the private school heads collected the survey questionnaire and researcher was asked to collect after two to three days. The return rate from the public sector was 100% and the private sector was 80% with overall return rate of 87%.

3.2. Phase Two

3.2.1. Reliability of the Mathematics Teacher Survey Questionnaire

The survey questionnaire reliability was measured by running Cronbach's Alpha on SPSS to measure the internal consistency of each item used in MTSQ. Table 1 shows the score of Cronbach's Alpha on the overall survey questionnaire estimated was 0.77 with alpha values of the sections ranging from 0.48 to 0.79 which demonstrates the scale as highly reliable.

Table 1. Internal Consistency Reliability Statistics

Sections	# of items	Cronbach's alpha
Nature of mathematics (Item 1 – 11)	11	0.48
Teaching and learning (Item 12 – 23)	12	0.79
Teaching strategies (Item 24 -37)	14	0.64
Overall survey questionnaire	**37**	**0.77**

3.2.2. Test of Normality

The Kolmogorov-Smirnov test was performed on all 37 items and the result shows $p < 0.0001$ which means the distribution of the sample is unlikely to be from a normal distribution. Nevertheless, exploratory factor analysis was used for the exploratory purpose to identify patterns and map teachers' beliefs about mathematics, teaching and learning of mathematics on reduced factors.

4. Results

4.1. Exploratory Factor Analysis

Exploratory factor analysis (EFA) was performed to reduce the data set to a few factors (Field, 2005), explore patterns (Cohen et al., 2000) and correlation among different variables to describe key aspects of teachers' knowledge about mathematics, mathematics teaching and learning. Moreover, it is recognized that the data was skewed so while drawing conclusions and interpretations researcher was cautious. The 'Kaiser-Meyer-Olkin' (KMO) measure of sampling adequacy and 'Bartlett's Test' of sphericity was run to see whether the sample size is appropriate for factor analysis and the strength of the relationship among the variables is significant (Blaikie, 2003). The Kaiser-Meyer-Olkin values for the questionnaire exceeded the recommended value of 0.6 and the Bartlett's Test of Sphericity for the dimensions also reached statistical significance i.e. value to be less than 0.05 (Field 2005) supporting the suitability of the data for factor analysis. Table 2 presents that KMO is 0.631 and Bartlett's test is significant [$\chi2$ (666) =1357.789, $p<0.001$].

Table 2. KMO and Bartlett's Test for the overall Survey Questionnaire

Kaiser-Meyer-Olkin Measure of Sampling Adequacy		0.631
Bartlett's Test of Sphericity	Approx. Chi-Square	1357.803
	Df	666
	Sig.	.000

Next the eigen values and scree plots were used as criteria for extracting factors that are statistically important. The factors with eigenvalues greater than 1 were retained. By plotting a graph of each eigenvalue (Y-axis) against the factor with which it is associated (X-axis) i.e. scree plot, the relative importance of each factor becomes apparent which helped in taking decision to retain the key factors. Varimax factor rotation method was used as it tried to load a smaller number of variables highly onto each factor resulting in more interpretable clusters of factors.

Table 3 presents the results using factor loadings in the extracted factors, means and standard deviations of the original item scores to explain the patterns in teachers' knowledge about the nature of mathematics.

4.1.1. Nature of Mathematics

The analysis of the nature of the mathematics scale as demonstrated in table 3 identified four factors extracted with the help of a scree plot. The four-factor solution explained 57.12% of the variance, with Factor 1 contributing 18.55%, Factor 2 contributing 16%, Factor 3 contributing 12.41%, and Factor 4 contributing 10.16%.

Looking at the items grouped under four factors in the Table 3, Factor 1 was titled 'Absolutist views about mathematical knowledge' because the items under this factor describe mathematics as a static discipline; Factor 2 'Source and uses of mathematics knowledge' because it explains different usage of mathematical knowledge in the real life, Factor 3 'Components and connections within mathematics' and Factor 4 'Human beings as mathematical knowledge constructors.' Table 4 presents the results using factor loadings in the extracted factors, means and standard deviations of the original item scores to explain the patterns in teachers' knowledge about the nature of mathematics.

Table 3. Item loadings - Nature of mathematics

Factors	Loadings	Mean	SD
Factor 1: Absolutist views about mathematical knowledge			
3. Mathematical rules can never be proved wrong.	0.752	3.44	1.068
5. Current mathematical knowledge will remain the same in the future.	0.685	2.93	1.161
9. Mathematical knowledge is the same throughout the world.	0.677	3.62	1.115
10. Study of mathematics is suited mostly to males.	0.536	2.32	1.213
Factor 2: Source and uses of mathematics knowledge			
2. Mathematics contributes to scientific inventions.	0.676	4.38	.652
6. Mathematics existed in the world even before human creation.	0.796	4.10	.897
11. Mathematical knowledge can contribute to addressing societal issues (e.g. inequality, environmental issues)	0.694	3.76	.881
Factor 3: Components and connections within mathematics			
1. Mathematics comprises only formulae, symbols and rules.	-0.674	3.48	1.231
4. Mathematical knowledge consists of several concepts which have connections among them.	0.788	4.12	.854
Factor 4: Human being as mathematical knowledge constructor.			
7. Mathematics is a creative subject like arts/music.	0.665	3.96	1.026
8. Human beings create mathematical knowledge.	0.788	3.85	.986

4.1.2. Teaching and Learning of Mathematics

The analysis of teachers' views about teaching and learning mathematics produced three factors. This three-factor solution explained 53.96% of the variance, with Factor 1 contributing 33.78%, Factor 2 contributing 11.19%, and Factor 3 contributing 8.96

Table 4 represent three factors, Factor 1 was titled 'individualistic and product-oriented mathematics learning, teaching and assessment' because the items under this factor explain different teaching and learning techniques that promote instrumental understanding; Factor 2 as 'collaborative learning promotes mathematical understanding'; and Factor 3 'perception about gender and technology'. Table 4 presents the results using factor loadings, means and standard deviation of the original item scores to explain the patterns in teachers' views about mathematical teaching and learning.

Table 4. Item loadings - Mathematics teaching and learning

Factors	Loadings	Mean	SD
Factor 1: Teaching and learning for procedural understanding			
12. Mathematics is learnt well by working individually on mathematical problems.	.595	3.32	1.176
13. Students learn mathematics by finding the correct answer to mathematical problems.	.688	3.42	1.188
14. Students can solve all mathematical problems on the basis of a memorized formula.	.766	3.18	1.319
15. The most effective way to teach mathematics is to first explain the topic, give example(s) and then ask students to solve mathematical tasks.	.681	4.11	1.006
17. The more students practice mathematical exercises given in the textbooks, they learn mathematics with understanding.	.734	4.11	.961
18. Teacher is the main source of mathematical knowledge.	.684	3.51	1.205
19. Textbooks are the main resource to teach mathematics.	.719	3.31	1.184
23. Paper pencil test is the best way to examine students' mathematical learning.	.549	3.32	1.178

Table 4 continued

Factors	Loadings	Mean	SD
Factor 2: Interaction promotes mathematics learning			
20. Students develop better understanding of mathematics when they work in small groups.	.743	4.26	.797
22. Real-life questions are an important component for teaching mathematics.	.759	4.23	.752
Factor 3: Perception about gender and technology			
16. Use of calculator hinders students' understanding of mathematics.	.402	3.39	1.170
21. Boys are better at learning mathematics than girls.	.940	3.10	1.317

4.1.3. Teaching Practice

Table 5 shows that in this section -'teaching practices' - four factors were retained. The four-factor solution explained 51.96% of the variance, with Factor 1 contributing 19.43%, Factor 2 contributing 15.13%, Factor 3 contributing 8.81%, and Factor 4 contributing 8.59%.

Looking at the items under these four factors, Factor 1 was titled 'promoting conceptual understanding'; Factor 2 'promoting procedural understanding'; Factor 3 'teaching practice promoting procedural understanding'; and Factor 4 'teaching practice promoting conceptual understanding'. Table 5 presents the results using factor loadings, mean score and standard deviation values to explain the patterns in teachers' mathematics teaching practice in terms of how often they used different techniques and strategies as reflected in the extracted factors.

Table 5. Item loadings - Teaching practice

Factors	Loadings	Mean	SD
Factor 1: Promoting Conceptual Understanding			
27. Students used concrete materials while learning mathematics. (such as blocks, beads, paper cutting and folding etc)	.557	3.06	1.164
28. Students used calculators while learning mathematics.	.445	2.46	1.185
29. Encouraged students to share their ideas about mathematical concepts when first introduced to them. (brainstorming)	.519	4.15	.916
32. Assigned mathematical problems which have more than one right answer.	.407	3.38	1.095
35. Assessed students' learning through quizzes.	.620	3.60	.880
36. Assessed students through project work.	.793	2.70	1.056
37. Engaged students in small group work.	.627	3.34	1.034
Factor 2: Promoting Procedural Understanding			
31. Engaged students to practice all mathematical exercises given in the textbooks.	.426	4.53	.769
33. Engaged students in individual work.	.724	4.01	.095
34. Assessed students' learning using paper-pencil tests.	.759	4.29	.856
Factor 3: Teaching Practice Promoting Procedural Understanding			
24. Used textbook-based tasks.	.423	4.21	.749
30. Explaining mathematics rules, formulas and procedures to students	.832	4.74	.489
31. Engaged students to practice all mathematical exercises given in the textbooks.	.575	4.53	.769
Factor 4: Teaching Practice Promoting Conceptual Understanding			
25. Engaged students in real-life problem-solving.	.795	3.60	.852
26. Used surroundings while teaching mathematics	.715	3.90	.859

5. Discussion and Conclusion

Understanding teachers' viewpoints is considered important because one of the key elements discussed generally in the teacher education literature is that what teachers bring with them, their conceived knowledge about the nature of mathematics has a great influence on what they do in classrooms (Barkatsas, 2008; Dossey, 1992; Halai, 2001; Gates, 2006; Lloyd, 2002). While developing the tool, special attention was given to the socio-cultural aspects, and this proved to be a significant contribution that makes the tool more relevant to the

Pakistani context. There are various perspectives through which philosophers have discussed the nature of mathematical knowledge. In general, the two dominant epistemological perspectives of mathematical philosophies are 'absolutism' and 'fallibilism' (Ernest, 1991; Jaworski, 1996; Lerman, 1990). At one extreme, mathematics is seen as static, fixed and either discovered or waiting to be discovered, i.e., 'absolutist view of mathematics' and at the other extreme mathematics is seen and interpreted as socially constructed phenomena, i.e., 'fallibilist view of mathematics'. Overall, the tool developed was able to capture these two major viewpoints that teachers possess as the items developed were relevant to their own context and experience as a learner and teacher of mathematics. To elaborate one key finding that Pakistani teachers hold dualistic views about the nature of mathematics i.e., they consider mathematics as both a discovered and an invented body of knowledge. The possible reasons of this dualistic viewpoint could be due to the socio-cultural and religious faith as Pakistan is an Islamic society where teachers believe in Almighty as 'knowledge creator'.

In addition, it was evident that most of the teachers view teaching as product-oriented where teachers transmit knowledge to students from textbooks, through lectures and teacher-directed questions and answer sessions (Dossey, 1992; Halai, 2001). It is likely that teachers' experiences of learning mathematics through transmission might have led them to perpetuate existing beliefs and perceptions instead of challenging them.

The tool development process provided evidence for validity and reliability based on analyzing the data collected from 200 practicing mathematics teachers teaching in Pakistani context. An exploratory factorial analysis has been carried out to ensure the internal consistency and one-dimensionality of the tool. The obtaining KMO coefficient has confirmed that the items in each theme make up one single factor.

Mathematics Teacher Survey Questionnaire (MTSQ) developed as part of the doctoral study being a valid and reliable survey questionnaire is available to teacher educators to explore teachers' knowledge about the nature of mathematics, their views about the teaching and learning of mathematics. Based on analyzing the teachers' responses to this survey questionnaire, teacher educators will be able to develop need-based intervention programmes to promote quality teaching and learning of mathematics in the country. For instance, if teachers strongly believe in mathematics is static body of knowledge, then implementing curriculum that demands constructivist learning approaches will not be achieved. Therefore, first teachers need to experience the beauty of mathematics and explore variety of ways to engage children in constructing mathematical knowledge rather believing that mathematical knowledge can be transmitted, and procedures are more important to be memorize rather than understood.

Funding: The study was funded by the Aga Khan University Institute for Educational Development– Doctoral Study Grant.

Declaration of interest: The author declares no competing interest.

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