Student-Teacher Experiences in Numeration Systems

Clement Ayarebilla Ali[a] and Michael Tangkur[b]

[a]University of Education, Winneba, Ghana
[b]SDD University of Business and Entrepreneurial Skills Development, Ghana (ORCID: 0000-0002-3268-6452)

Article History: Received: 13 January 2024; Accepted: 13 March 2024; Published online: 26 April 2024

Abstract: There are fundamental relationships among numeration systems. However, student-teachers fail to adequately exploit the fun and flair of relationships in numeration systems. In this study, we explored the didactical phenomenology experiences to bridge this gap. Even though the sampling technique was purposive, the sample size of 16 groups of 10 student-teachers per group was determined by the principle of saturation, as no new lived experiences of participants added any new information on the themes. Focus group discussions were used to collect data. The 16 groups were administered the same set of questions. After discussions with each group, their responses were recorded, transcribed, coded, and analyzed to reveal five themes. Thereafter, a phenomenological analysis was employed to describe their experiences. The analysis showed that student-teachers could readily apply the methods of conversions to base 10. However, they found it very hectic to convert to other bases. It was therefore recommended that much more didactical phenomenology experiences should be exploited to teach number bases to student-teachers.

Keywords: Lived experiences, Numeration systems, Phenomenological experiences, Student-teachers

1. Introduction

Numeration systems have existed throughout human history. Numeration systems are structured methods or procedures for counting to determine the total units in a collection. This consists of counting bases (Base 2, Base 5, Base 4, Base 6, Base 8 and Base 10. These systems sought to identify people, preserve confidentiality, increase security, and minimize errors (Encyclopedia.com, 2018).

There exist fundamental relationships among the numeration systems and human life experiences. For instance, it has been amply demonstrated (Edara, 2017; Houdement & Tempier, 2019; Wadhwa, 2023) that base ten systems were developed because of ten fingers, or digits, base 2 for 2 groups of units, base 3 means 3 groups of units, base 6 means 6 groups of units, and base 12 means 12 groups of units. All these numerations were generated as a result of human experiences with their body parts and counting.

The conversion of bases explores the concept of 'Powers of Numbers'. Powers of Numbers describes the number of times a base multiplies by itself. For instance, 2^3 means '2' raised to the power of '3'. This is because place value in the decimal Hindu-Arabic system was based on two inseparable concepts. These were the positional principle and the decimal principle. The positional principle explains the position of each digit in a number and the decimal principle explains the number of ten units below or above a given digit (Wadhwa, 2023). For instance, in the number '1,234', the position of '4' is units, the position of '3' is tens, the position of '2' is hundreds, and the position of '1' is thousands. By positional principle, '3' is greater than '4', '2' is greater than '3', and '1' is greater than '2'. By decimal principle, ten times of units shift to tens, ten times of tens shifts to hundreds, and ten times of tens shifts to thousands. Therefore, there is so much 'power' accorded to tens over units, hundreds over tens, and thousands over hundreds. Could this 'power' be linked to the 'Power of Conversion'?

In place value, numbers occupy different positions and represent ordered values of different magnitudes. The value of each position is determined by base-10 rules (e.g., the rightmost position is 10^0, followed by 10^1, 10^2, and so on. Brown (2009) has made similar allusions to the 'power' of extracting and making transformations. This possibility is the core of transforming the various number bases. Each digit has a place value that determines the value of that digit according to its position in the number. The value of a digit in a number increases as we move the digit from left to right, and decreases as we move the digit from right ~~left~~ to left ~~right~~. So, the digits on the left have a lower place value than the digits on the right.

In computing, however, we generally only come across base 2 (binary), base 8 (octal), base 10 (denary), and base 16 (hexadecimal) (Tangarajah, 2022). In everyday life, base 5, base 6, base 7, and base 9 have practical cultural scenarios. These include but are not limited to the five working days, six months in a half year, seven days in a week, and nine months for a woman to deliver.

The power of imagination (Brown, 2009) is strong enough to transform any of the number system categories. It is just unfathomable to decipher the transformation from units to tens, to hundreds, and thousands. This can be related to inexplicable powers in the family, the house, and the society. These are powers accorded to human by their cultures and we simply describe this phenomenon as 'cultural powers'. But how can this 'cultural power' be faded into number bases? We can observe that by converting other bases to base ten, we must first consider

Corresponding Author: Clement Ali **email:** aliclement83@gmail.com

Citation Information: Ali, C. A. & Tangkur, M. (2024). Student-teacher experiences in numeration systems. *Turkish Journal of Mathematics Education, 5*(1), 66-73.

the positional notation. In converting base ten to other bases, we have two cases. **Case 1 has numbers carrying no fractional part. So, we use the** division method. **Case 2 has numbers carrying the real part and the fractional part**. So, we use both division and multiplication methods. Another method for case 2 is to use the highest power of the base that will divide into the given number.

Many research works (Bos, Doorman & Piroi, 2020; Larsen, 2018; Loc &Tien, 2020) bequeath us with a strong theory to make these transformations effective and factual. This is the theory of Realistic Mathematics Education which emerged with guided reinvention, emergent models, and didactical phenomenology. The emergent model's design principle explains how students' activity at several levels is part of a reinvention process but does not necessarily require rigorous immediate results (Bos, Doorman & Piroi, 2020). Guided reinvention is an instructional design that creates mathematics instruction that would result in students recreating mathematics concepts with careful guidance from the teacher. In didactical phenomenology, the description of phenomena, facts, and circumstances produces mathematical concepts, structures, or ideas (Loc & Tien, 2020). But does phenomenology relate to the power and culture of numbers?

The power of children to understand their teachers cannot be underrated. The various methods of manipulation, abstractions, and sharing play a major role (Brown, 2009). The study concentrates on only student-teachers' verbal descriptions and understanding of the inter-conversion of the number bases. Da (2022) perfects this path with natural feels and urgencies. By reading and analyzing their statements from the transcripts, we reflected on the teaching methodologies that consequently guided the research questions of the study:

1) What methods do student-teachers use for converting number bases?
2) How do student-teachers experience conversion of number bases?

2. Materials and Method

As Brown (2009) agrees, the teacher can explain all phenomena under their power but what matters is whether the student has been answered. In this direction, phenomenology was chosen to help gather information on how individual student-teachers experience the conversions of number bases and how they feel about changing number bases for learning to take place.

There is still a bone of contention on the ideal sample size of a qualitative phenomenological study. However, the point of saturation determined the satisfaction of the data collected. At saturation, the collection of new data would not have added information to the phenomenon under investigation (Vasileiou et al., 2018). With a class of 160 student-teachers at our disposal, the researchers made 16 groups and each group contained about 10 participants. We believed that this was enough to attain the point of saturation. The sampling selection technique was purely purposive. So, we selected the participants based on their experiences, institutional memory, specificity, purpose, and relevance in the study. This minimized domination based on gender, intelligence quotient, previous knowledge, and performance (Sebele-Mpofu, 2020).

We used focus group discussion to collect the data. A study by Sebele-Mpofu (2020) suggests that 6 to 8 interviews are enough when researching a homogenous sample. Despite different arguments, the interviews adequately rationalized the claims and conclusions drawn by the researchers. The researchers formed 16 groups, and each group was administered the same set of seven questions. After discussion with each group, the responses were recorded, transcribed, coded, and analyzed in themes.

In the data analysis, **interpretive phenomenology** was used to connect the participants' knowledge **in number bases and conversion**. The phenomena ranged from merely understanding the concept of number base **to extremely** rare complicated personal experiences of converting numbers with decimal points (Warren, 2020). Every statement was interpreted by the phenomenon.

The internal ethical considerations required that we use pseudonyms for all participants to ensure the participants have a level of confidentiality for the study. All recordings were under the researchers' care, and passwords were secured on our private computers. Voice recordings were made with password protection, and stored in a secret folder with the only keys in the personal possession of the researchers. Participant consent forms were accompanied, to ensure ethical considerations or implications of the research.

The participants were instructed to participate in the research on a volunteer basis. The research can be terminated at any time should any participant wish to do so. Any other unforeseen occurrences that could jeopardize the participants in any way were not wanted or desired by the researchers, and all precautions were maintained.

3. Results

In this section, five themes were identified. Each theme consists of at least five questions and their response. The responses are transcriptions of the pupils' worksheets of the tasks and are summarised for easy understanding. After every question, a short comment is provided.

3.1. Theme 1: Methods of Converting Other Bases (two) to Base Ten

In this theme, we sought to explore the student-teacher's knowledge and understanding of converting any bases to base 10. This would help us to discover the errors therein and help reduce them. The responses of the groups were presented in Table 1.

Table 1. The responses of the groups regarding the first question

Question 1: How do you convert the numerals to base 10?	
Group 1	$10.011_2 = (1\times 2^1)+(0\times 2^0)+(0\times 2^{-1})+(1\times 2^{-2})+(1\times 2^{-1}) = 2.375$
Group 2	$1.101_2 = (1\times 2^0)+(1\times 2^{-1})+(1\times 2^{-2})+(0\times 2^{-3}) = 1.625$
Group 3	$10.011_2 = (1\times 2^1)+(0\times 2^0)+(0\times 2^{-1})+(1\times 2^{-2})+(1\times 2^{-1}) = 2.375$
Group 4	$10.001_2 = (1\times 2)^1+(0\times 2)^0+(0\times 2)^{-1}+(0\times 2)^{-2}+(1\times 2)^{-3} = 2.125$

The four groups could adequately express numbers in Powers of Base 10. This strategy is common in the high school textbooks and other state-approved mathematics books. In each of the expressions, we can observe both the positional and the decimal principles fully utilized. However, Group 4 committed of error of making the brackets to affect both the multipliers and multiplicands.

3.2. Theme 2: Methods of Converting Base Ten to Other Bases (Two)

In this theme, we sought to delve deep into the reversibility thoughts of converting number 10 numerals to other bases as exemplified by base 2. This would help them relate the two-way route of conversion. This connection and relational learning could extend future relationships. The responses of the groups were presented in Table 2.

Table 2. The responses of the groups regarding the second question

Question 2: How do you convert the following numerals from base 10?	
Group 5	$1032.6875 = (8^3)+(8^3)+(8^1)+5\times(8^{-4})+3\times(8^{-2})+3\times(8^{-1}) = 2010.6651_8$
Group 6	$172.878 = 2\times(8^2)+5\times(8^1)+(4)+(8^{-3})+5\times(8^{-2})+5\times(8^{-1})+6 = 260.551$
Group 7	$25.125 = (2^4)+(2^3)+(2^0)+(2^{-6})+(2^{-5})+(2^{-4})+(2^{-3})+(2^{-2})+(2^{-1})+(2^0) = 11001.0111111_2$
Group 8	$172.872 = (8^2)+(8^2)+(8^1)+(8^1)+(8^1)+(8^1)+(8^1)+(8^{-3})+6(8^{-2})+5(8^{-1}) = 254.551_8$

The four groups could adequately convert numbers from base 10. This strategy is familiar to student-teachers. However, its scope and content are much more difficult than the conversion to base 10. Apart from Group 5 which correctly expressed both the base and the number, all other groups skipped the multipliers. The consideration was much more based on the bases and their exponents.

3.3. Experiences in Converting from Other Bases to Base Ten

In this theme, we sought to know and understand the errors and misconceptions student-teachers might have faced in converting the number bases in Themes 1. If we know and understand the real challenges in words and speech, we could better address the issues as compared to the written responses alone. The responses of the groups were presented in Table 3.

Table 3. The responses of the groups regarding the third question

Question 3: What is your experience in converting numerals to base 10?	
Group 9	Conversions from other bases to base 10 are NOT difficult. This is because, in conversions from other bases to base 10, you will just multiply the number base using their base values. So after writing the place values starting from the right side, you will need to multiply each digit by its corresponding place value and then you will add the products.
Group 10	Not difficult. This is because the base-ten numeration system is commonly used everywhere for simple calculations of hundreds, tens, and ones. Knowing the base you are working with, you only need the product of the exponential notation of that base and multiply it by their respective place value digit, which makes it much simpler.
Group 11	Conversions from other bases to base ten are NOT difficult because you first of all write the place value starting from the right-hand side. And you write each digit under its place value. Also, you multiply each digit by its corresponding place value and lastly, you add up the product. But when the question involves decimals, decimals students find it difficult to locate the lace value and often find it hard to simplify the numerals after the decimal point using the power expansion method.
Group 12	No! It is NOT difficult because you will have to use the base to multiply the figures in the number given. For example, when you are given a number like 10111 to convert to convert to base 10 this is how you will do it, making it easy to get the answer.

The four groups eloquently alluded that it was NOT difficult to convert bases. However, the practice of converting the bases is not adequate. Many student-teachers use unspecified and shortcut strategies to obtain the answers. Such strategies are obnoxious because they do not provide scientific proof in mathematics.

3.4. Experiences in Converting Base Ten to Other Bases

In this theme, we sought to unearth and discover the real challenges in converting base 10 numerals to other bases. The conversion in Theme 2 presented a bigger challenge. With succinct and real-life verbal cues and expressions, we could empathize with and assist them with their anxiety. The results were summarized in Table 4 and Table 5 respectively.

Table 4. The responses of the groups regarding the fourth question

Question 4: What is your experience in converting from base 10 to other numerals?	
Group 13	Difficult! This is because learners must first know the highest exponents of the bases they are working with. This is confusing in such a way that others may confuse and exchange the exponents with factors of the bases they are asked to work with.
Group 14	No but at times yes! Yes, it is difficult because first, you have to break down the number to the smallest divisible number before you can be able to use the exponent to find the answer. So, if a student does not know a number exponent a number he/she won't be able to solve it.
Group 15	To convert any base in base 10 to any other base, we use basic division with remainders. Do not divide using decimals, the algorithm will not work. Keep dividing by until your quotient is zero. Now you will write your remainder backwards. That makes the conversion from base 10 to other bases difficult.
Group 16	Difficult! Finding the highest power may be confusing, sometimes we exceed the highest power given and end up having figures greater than expected. Learners find it difficult to find the roots of numbers (powers of numbers with their corresponding figures).

The four groups agreed that it was a very difficult exercise. Many more strategies are essential to boost their understanding and improve upon their learning of number-based conversions. In this context, so many more tutorials were required to extend and consolidate your knowledge and competencies.

Table 5. The responses of the groups regarding the fourth question

Question 5: Are conversions from other bases to base ten difficult more than conversions from base ten to other bases? Explain your answer.	
Group 1	Yes, because converting the number base ten to the other bases; first you have to divide the number by the base to get the remainder. This remainder is the first i.e. least significant digit of the new number in the other base. Again, you have to repeat the process by dividing the quotient of step 1, by the new base. This time, the second remainder is the least significant. Also, you have to repeat the process until your quotient is less than the base. This quotient is the last digit, i.e. the most significant digit.
Group 2	No. Because conversions from other bases to base ten are NOT difficult at all. Converting between other bases to base ten is fairly as long as you remember that each digit in the other base number represents a power of that base number.
Group 3	Conversions from other bases to base ten are difficult even as compared to conversions from other bases to base ten. This is because in conversions from base ten to other bases, one might meet larger digits that might not be easy to comprehend and that even in the absence of a calculator one must solve. Nonetheless, conversions from other bases to base ten are NOT difficult as compared to conversions from other bases to base ten in the sense that, as elaborated earlier, one must just take note of every digit and the exponents that accord it so as one would not have wrong answers or results.
Group 4	No. Because the Hindu-Arabic numeration system has provided for the base 10 to other bases is more difficult than converting other bases to base 10. Pupils mostly or always deal with the decimal system that is 0-9, the base 10 system, they usually feel more comfortable when converting numbers with other bases to base 10, than when converting from base 10 to other bases.
Group 5	The conversions from base ten to other bases are rather difficult than the conversions from other bases to base ten. Base 10 comprises all other bases (0,1,2,3,4,5,6,7,8,9). Yes, it is difficult. The conversion from base 10 to the other bases increases the digits when compared to base 10 numerals even though they are of the same value. Even for base 2, the base 2 answer becomes a lot (having more digits). And it is also for the same reasons, which makes conversions from base 10 to other bases more difficult.
Group 6	No because learners are more familiar with the addition and multiplication of numbers but find it difficult when it has to do with finding the factors of a given number.

Table 5 continued

Group 7	Conversion from other bases to base 10 is not difficult because of the answer that you will get when you change it to other bases. Again, you will get the same answer but from base 10 to other bases, the decimals will give you different options.
Group 8	Yes, because converting from base ten to other bases is difficult when you consider converting from base 10 to base 8 is more confusing and you will find it difficult to know how to convert it.
Group 9	No! Conversions from other bases to base 10 are easier than conversions from base 10 to other bases. This is because, when changing from other bases to base 10, dividing the exponents to work with is easier.
Group 10	The conversions from base ten to other bases are rather more difficult than the conversions from other bases to base ten. Base 10 comprises all other bases (0,1,2,3,4,5,6,7,8,9). Yes, it is difficult. The conversion from base 10 to the other bases increases the digits when compared to base 10 numerals even though they are of the same value. Even for base 2, the base 2 answer becomes a lot (having more digits). And it is also for the same reasons, which makes conversions from base 10 to other bases more difficult.
Group 11	We stand for 'NO'. Conversions from other bases to base 10 are NOT difficult than conversions from base 10 to other bases because, converting other bases to base 10 you only need to remember that each digit in the other base number represents a power of in the other base number represents a power of that and apply the use of place value.
Group 12	No. It is NOT difficult because you will have to use the base to multiply the figures in the number given.
Group 13	Conversions from other bases to base ten are NOT but conversions from base 1o to other are difficult because the procedure used in solving from other bases to base 10 is simple. A learner needs to know the powers allotted to each digit at the right corner. If it is a whole number but if it includes decimals you indicate that particular digit after the decimal point with negative numbers but not exponents. After learners can continue with the rest of the procedures by multiplying and adding the numbers obtained together to get an answer, unlike working from base 10 to other bases, this is very difficult and complex highest exponent of the bases assigned to you as a learner needs to be found and written down so you have no more exponent left. After the exponent of each number is to be written under it, Then you add zero to one depending on the exponent you are working with.
Group 14	No. Conversions from other bases to the base are not difficult because you just multiply the base given by the number in other to get the base 10 numeration.
Group 15	So with the two examples, changing from other bases to base 10 is NOT difficult compared to conversion from base 10 to another base is difficult. In conclusion, changing from other bases to base 10 is so simple, not difficult at all but changing base 10 to the other bases is very difficult.
Group 16	No! Conversions from other bases to base 10 are not as difficult as conversions from base 10 to other bases. Conversions from base ten to other bases do not demand a lot of critical thinking in finding the exponents as compared to from base 10 to the other bases.

It was very clear that the groups had mixed reactions. However, the majority believed that it is more difficult to convert to other bases than to convert to base 10.

4. Discussion

In this discussion, didactical phenomenology helped the researchers to create contexts and tasks that can promote the emergence of targeted number bases as models for student-teachers informal activity. This was well situated in the theoretical framework. In each theme, the analysis allowed us to pose tasks that begged to be addressed using informal strategies that anticipated the group concept (Larsen, 2018). The discussion of the findings is based on converting other bases to base 10, converting base 10 to other bases, and converting other bases to other bases.

4.1. Converting other bases to base ten

Converting any base to base 10 is fairly simple, as long as one remembers that each digit in the base number represents a power of the base. For instance, in converting 101100101_2 to the corresponding base-ten number, we just list the digits in order. For instance, in converting *101100101_2* to base 10, we have two rows for digits and numbering. Then, in another row, we count these digits off from the right, starting with zero:

Thus, $1\times2^8 + 0\times2^7 + 1\times2^6 + 1\times2^5 + 0\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0 = 1\times256 + 0\times128 + 1\times64 + 1\times32 + 0\times16 + 0\times8 + 1\times4 + 0\times2 + 1\times1 = 256 + 64 + 32 + 4 + 1 = 357$. Then *$101100101_2$ convert to 357_{10}*. This is the most common conversion as every culture now reverts to base 10. The experiences of the student-teachers have found this conversion to be the easiest and simplest way of representing numbers and numerals at any level.

However, student-teachers most likely skip some places. Others may forget to use the multipliers as witnessed in Themes 1 and 2. Larsen (2018) suggests a table entailing the places and the values to reduce such errors during manipulations.

4.2. Converting base ten to other bases

Converting decimal numbers to any base is nearly as simple: just divide by the base and keep the remainder. For instance, in converting 357_{10} to the corresponding binary number, we need to divide repeatedly by 2, keeping track of the remainder. In converting a base 10 with a decimal point to any base, for instance, **$(172.878)_{10}$ to base 2,** we treat the real part and fractional part separately. For the **real part,** we convert the real part from base 10 to base 2 using the division method. So, $(172)_{10} = (10101100)_2$. **For the fractional part,** we convert the fractional part from base 10 to base 2 using multiplication methods. In Table 6, the fractional part does not terminate at 0 after several iterations. So, let us find the value up to 4 decimal places. Traverse the real part column from top to bottom to obtain the required number in base 2. From here, $(0.878)_{10} = (0.1110)_2$. Combining the results of a real part and a fractional part, we have $(172.878)_{10} = (10101100.1110)_2$.

Table 6. Converting base ten to other bases

Operations	Real part	Fractional part
0.878 × 2	1	0.756
0.756 × 2	1	0.512
0.512 × 2	1	0.024
0.024 × 2	0	0.048

Another method of converting a base 10 number to any base utilizes the powers of the base to be converted. For instance, in converting the base 10 number 63201_{10} to base 7 can be found as follows: the highest power of 7 that will divide at least once into 63201 is 7^5. When we do the initial division, we would obtain $63201 \div 7^5 = 3.760397453$. Here, the decimal part is non-terminating. To avoid an error, we leave the result and simply subtract 3 to get the fractional part all by itself. Subtraction and then multiplication by seven gives:

- $63201 \div 7^5 = 3.760397453$
- $0.760397453 \times 7 = 5.322782174$
- $0.322782174 \times 7 = 2.259475219$
- $0.259475219 \times 7 = 1.816326531$
- $0.816326531 \times 7 = 5.714285714$
- $0.714285714 \times 7 = 5.000000000$

At long last, the last product is exactly 5. This gives us our final result as $63201_{10} = 352155_7$. Remember that if the first division is by 7^5, then you expect to have 6 digits in the final answer, corresponding to the places for 7^5, 7^4, and so on down to 7^0. If you find yourself with more than 6 digits due to rounding errors, you know something went wrong.

The student-teachers found this conversion to be more tedious and challenging than the conversion to base 10. The conversion is even much more challenging if there is any decimal number to be converted. The dual issue of solving simultaneously the whole and the decimal parts posed complex routes to the solutions.

4.3. Converting other bases to other bases

In converting a base to another base other than base 10, say, base 2 to base 8, we need to group the binary digits from the right in groups of three. A common example is converting 111101001011_2 to base 8:

111	101	001	011
7	5	1	3

In the base 8 conversion, we are required to group the digits three at a time, starting from the right. Therefore the binary number 111101001011_2 is the octal number, 7513_8

Sometimes, the number of digits in a binary number may not be exactly divisible by 3 or 4. In such situations, you may start grouping the digits three at a time and finish with one or two 'extra' digits on the left side of the number. In this case, simply add as many zeros as you need to the front (left) of the binary number. Let us consider converting 1010_2 to base 8. We start by adding two zeros to the front of the number to make it 001010_2. We now have six digits, which can be conveniently grouped three at a time. With this change, 001010_2 can be easily converted to 12_8.

001	010
8	2

In converting Binary to Octal, the following steps are necessary:

- Step 1 − Divide the binary digits into groups of three (starting from the right).
- Step 2 − Convert each group of three binary digits to one octal digit.

Therefore, the Binary Number, 10101_2 is the Octal Number, 25_8

Conversely, the following table is an example of converting an octal number to a binary number:

- Step 1 − Convert each octal digit to a 3-digit binary number (the octal digits may be treated as decimal for this conversion).
- Step 2 − Combine all the resulting binary groups (of 3 digits each) into a single binary number.

Therefore, the Octal Number, 25_8 is the Binary Number, 10101_2

An alternative to this method can be obtained from the following steps:

1. Step 1 − Convert the original number to a decimal number (base 10).
2. Step 2 − Convert the decimal number so obtained to the new base number.

For instance, in converting the Octal Number, 25_8 to a binary number:

- Step 1 − Convert to Decimal

Table 7. Whole number conversion of base 8 to base 10

Step	**Octal number**	**Decimal number**
Step 1	25_8	$((2 \times 8^1) + (5 \times 8^0))_{10}$
Step 2	25_8	$(16 + 5\ )_{10}$
Step 3	25_8	21_{10}

The Octal Number, 25_8 is the Decimal Number − 21_{10}

- Step 2 − Convert Decimal to Binary

Table 8. Whole number conversion of base 8 to base 2

Step	**Operation**	**Result**	**Remainder**
Step 1	21 / 2	10	1
Step 2	10 / 2	5	0
Step 3	5 / 2	2	1
Step 4	2 / 2	1	0
Step 5	1 / 2	0	1

Therefore, the Octal Number, 25_8 is the Binary Number, 10101_2 (Tangarajah, 2022).

Second, didactical phenomenology helped the researchers to create tasks that can promote the transition of these informal ideas into *models for* more formal number-based conversions. This was illustrated in the tasks student-teachers solved and narrated their experiences. In these two ways, didactical phenomenology provided the framework for the transitions into conversions (Larsen, 2018).

5. Conclusion

The didactical phenomenology provided the lived experiences of the student-teachers. The five themes originated from the student-teacher cultures. These included methods of teaching number bases, conversions to base 10, conversion from base 10, and conversion to other bases were the groundwork for the interpretations of the research. The most significant findings of this study were the conversion to base 10 and the conversion from base 10.

In Themes 1 and 2, we first discovered based on the student-teachers abilities to accept their feedback before any task would transpire, resulting in the student-teachers effort to reduce, continue, or maintain the task as pertained in their cultures. The feedback gave the student-teachers different outcomes to follow based on the use or non-use of the methods of conversions. After the feedback, the second interpretation emerged resulting in the conversion of other bases to base. The interpretation of the implications of receiving feedback only happened if the student-teachers had successfully converted their tasks from base 10 to other bases. These helped the student-teachers to reduce the errors to appreciable levels and built their knowledge and understanding of the conversions of number bases.

In Themes 3 and 4, we provided the student-teachers with alternative didactical routes. The choice of the implications of didactical phenomenology in converting to any base happened based on enacting or describing the process. The change was observed in the statements, comments, and descriptions. The student-teacher interpretation was discovered if the student-teachers confirmed the first two methods. The level of competency in different groups helped to make a new impact on their performance measures. Thus, leading to an action to transpire, creating new performance measures to emerge.

In Theme 5, we provided lived experiences to the student-teachers. These further addressed the concerns of the student-teachers leading to a discovery of new information based on their lived experiences. The examination of the short comments called for a change in the current format of converting base 10 to other number bases and addressing a yawning current gap in the literature.

We therefore recommended that the student-teachers' practices should not only aim at closing gaps in literature but also, to allow for research to be validated and scrutinized in place value and number bases in their cultural realms. We, therefore, hope to find the outcomes that would enhance student-teachers understanding of the synergies between culture and conversions of number bases. This would further invite more didactical phenomenology between culture and number bases.

Acknowledgments: We appreciate the participation of the student-teachers. Without their active involvement and interest in this project, the outcomes would have been a mirage. The second appreciation goes to the two authors who contributed to the conception and design of this study. It has been a long learning process.

Declaration of interest statement: We declare that apart from published works that have been duly cited, the authors bear NO financial, personal, or religious interests in the findings of this study.

References

Agape Bible Study (2023). *The significance of numbers in scripture*. Agape Bible Study, available at https://www.agapebiblestudy.com/documents/The%20Significance%20of%20Numbers%20in%20Scripture.htm.

Bos, R., Doorman, M., & Piroi, M. (2020). Emergent models in a reinvention activity for learning the slope of a curve. *The Journal of Mathematical Behavior, 59*(1), 1-15. https://doi.org/10.1016/j.jmathb.2020.100773

Brown, L. (2009). Spirituality and Student-Generated Examples: Shaping Teaching to Make Space for Learning Mathematics. In S. Lerman and B. Davis (eds.), *Mathematical Action & Structures of Noticing: Studies on John Mason's Contribution to Mathematics Education*, 147–160, Sense Publishers. https://doi.org/10.1163/9789460910319_013

Da, N. T. (2022). Approach to realistic mathematics education in teaching calculus for high school students: A case of the application of derivatives. *International Journal of Professional Development, Learners and Learning*, *4*(1), 1-9. https://doi.org/10.30935/ijpdll/11832

Edara, I. (2017) Religion: A Subset of Culture and an Expression of Spirituality. *Advances in Anthropology*, *7*(1), 273-288, https://doi.org/10.4236/aa.2017.74015

Houdement, C., & Tempier, F. (2019). I understand place value with numeration units. *ZDM Journal of Mathematics Education*, *51*(1), 25–37.

Encyclopedia.com (2018). *Numeration Systems*. Centigage. Available at https://www.encyclopedia.com/social-sciences-and-law/economics-business-and-labor/businesses-and-occupations/numeration-systems#C

Larsen, S. (2018). Didactical phenomenology: The engine that drives realistic mathematics education. *For the Learning of mathematics*, *38*(3), 25-29.

Loc, N. P., & Tien, N. T. T. (2020). Approach to realistic mathematics education in teaching mathematics: A case of cosine theorem–Geometry 10. *International Journal of Scientific and Technology Research*, *9*(4), 1173-1178.

Tangarajah, P. (2022). *Number bases*. Mount Royak University, retrieved from https://math.libretexts.org/Courses/Mount_Royal_University/MATH_2150%3A_Higher_Arithmetic/7%3A_Numeration_systems/7.2%3A_Number_Bases#:~:text=A%20number%20base%20is%20the,whole%20number%20greater%20than%200

Vasileiou, K., Barnett, J., Thorpe, S., & Young, T. (2018). Characterising and justifying sample size sufficiency in interview-based studies: systematic analysis of qualitative health research over a 15-year period. *BMC medical research methodology*, *18*, 1-18.

Wadhwa, P. (2023). *Expanded form of a number: definition, steps, value charts, examples*. EMBIBE. Retrieved from https://www.embibe.com/exams/expanded-form-of-a-number/

Warren, K. (2020). Qualitative data analysis methods 101: The big 6 methods+ examples. *Grad Coach*.