

Investigation of 7th Grade Students' Awareness of Verbal Problems with Missing Information or Excess Information

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Abstract: When the skills of mathematics education curriculum are considered, problem solving skill has an important place. In the development of students' problem solving skills, students need to examine and analyze the structure of the problem at the problem understanding stage. Encountering different types of problems with missing information and excess information will support students' ability to express their own thoughts and communicate mathematically. In addition, such problems provide students with opportunities to demonstrate their reasoning and creative thinking skills. The aim of this study is to examine 7th grade students' awareness of verbal problems with missing information or excess information. Case study method was used in the study. The sample of this study consists of 15 7th grade students studying in a public secondary school in Şanlıurfa province in Turkey. The students were selected by purposive sampling method and interviewed. A form consisting of a total of 6 problems, three of which contain missing information and three of which contain too much information, was used as a data collection tool. The data obtained were analyzed and interpreted descriptively. As a result of the study, it was seen that the students tended to solve the problem directly without determining what was given and what was required. This situation made it difficult for them to recognize missing and excessive information in the question. It was observed that they did not think about the problem, did not examine the data in the problem and were result-oriented. It was observed that the majority of the students had difficulty in recognizing verbal problems with missing information, but they were more successful in recognizing verbal problems with excess information. Since it is common to think that students have enough information to solve problems, it should be ensured that problems with missing and excess information are given in mathematics teaching and that they think comprehensively while examining the problems.

Keywords: Mathematics Education, Problem solving, Missing and excess information

1. Introduction

When the Mathematics Curriculum in Turkey is examined, it is seen that procedural knowledge was used more widely until 2005, but considering the importance of problem solving in human life, the curriculum in which procedural knowledge and conceptual knowledge are structured together was adopted in 2005 (Temiz and Çimen, 2017). Although the secondary school mathematics curricula published from 2005 to 2018 have undergone radical changes in their objectives, content, outcomes, skills, and implementation steps, "problem solving skill" is common to all curricula (İlhan and Aslaner 2019). The 2009 and 2013 mathematics curricula emphasized problem-solving skills and strategies, while the 2017 curriculum emphasized students who can solve problems (Şen, 2017).

In 2018, the specific objectives of the Mathematics Curriculum are grouped under thirteen items. Some of the items are stated as "will be able to easily express their own thoughts and reasoning in the problem solving process, see the deficiencies or gaps in the mathematical reasoning of others; will be able to develop metacognitive knowledge and skills, manage their own learning processes consciously; will develop a self-confident approach to mathematical problems by developing a positive attitude towards mathematics through their experiences in learning mathematics" (MoNE, 2018). Based on these items, it is seen that today's mathematics teaching aims to enable students to comprehend and make sense of the problem they will face, to choose the appropriate strategy for solving the problem, and to acquire problem solving skills at a level that can be applied in daily life situations. At the same time, based on the competencies adopted by the European Union Commission, it is aimed to raise individuals who understand the problem they face, can develop a solution and adapt it to different situations, have a positive attitude towards mathematics, and transfer mathematical problem solving to their daily lives, rather than just four operations skills.

In the problem solving process, students' tendency towards rote memorization and repetition, making mathematics based on four operations, and seeing it as a set of abstract operations cause them to exhibit negative attitudes towards mathematics (Çiçek, 2022). In order to avoid these situations, it is seen that students should first understand the problem. Understanding the problem is at the beginning of the problem solving process (Karataş and Güven, 2003). At the stage of understanding the problem, students are expected to determine what is given and what is required, what is missing or what is given too much (Erümit, 2014).

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Students cannot propose a solution for problems that they have difficulty in understanding, and they cannot reach a solution by applying a specific strategy. Perceiving mathematical knowledge and establishing a relationship between the information they make sense of occur in the problem solving process (Swings and Peterson, 1988). Students are expected to understand and interpret the problem they read in the mathematics course, express what is given and desired in the problem, and develop appropriate solutions to the problem (MoNE, 2009). According to Keşan et al. (2008), the deficiency in mathematical reading-comprehension and expression skills stems from inadequacy in expression skills.

The basic behavior expected from students at the problem comprehension stage is to recognize what is given, determine what is required, establish the necessary relationship between what is given and what is required, understand missing or excess information, and determine the necessary information (Gürsoy et al. 2015). It is thought that it is easier for the student who determines all these situations to determine the necessary strategy afterwards (Gökkurt Özdemir et al. 2018). It is inevitable that the student who does not determine the missing or excess information in the comprehension phase of the problem will get the solution wrong.

When the literature was examined, it was seen that studies were conducted on students' determination of missing or excess information or solving the problem with existing information. The studies consist of different grade levels. In the studies conducted in Turkey, 5th grade (Temiz and Çimen, 2017), 6th grade (Yenilmez and Yaşa, 2007) and 8th grade students (Gökkurt Özdemir et al. 2018, Çetinkaya and Soybaş 2018, Aydın and Özmen, 2012) were selected as samples, but no study was conducted with 7th grade students. Similar results were obtained in the studies and it was observed that students were more unsuccessful in recognizing missing information than in recognizing excess information (Çevik and Gökkurt-Özdemir 2018; Gürsoy et al. 2015; Soylu and Aydın 2006; Reusser, and Stebler 1997; Verschaffel, De Corte, and Lasure 1994). Similarly, it was found in similar studies that students had difficulty in identifying questions with missing and excessive information, did not question the information sufficiently, and did not pay attention to whether the given information was necessary for the solution (Temiz and Çimen, 2017; Gürsoy et al. 2015; Aydın and Özmen, 2012; Soylu and Aydın 2006). In their study, Low et al. (1994) concluded that providing students with text editing training is effective in determining the deficiency in the problem and how to eliminate the deficiency, and especially facilitates the solution of problems containing irrelevant information. Considering the studies, it is thought that examining 7th grade students' awareness of verbal problems containing missing information or excessive information is important enough to contribute to the literature. Considering that the problem solving success of the students is related to their success in understanding the problem and that equation setting and equation solving starts at the 7th grade level, the results obtained in this study will contribute to the field education.

1.1. Aim and Problem of the Study

In the study, it was aimed to examine the awareness of 7th grade students in problems with missing or excess information in the given problems. In the study, it is also important for the research to create awareness in the problem solving process by asking them to interpret which information can solve the problem if they realize the missing information or which information is too much in questions containing too much information.

In line with the aims of the study, the problem of the research was determined as "What are the awareness levels of 7th grade students about verbal problems containing missing information or excess information?". The sub-problems are;

1. What are the levels of 7th grade students' awareness of missing information in verbal problems?
2. What are the 7th grade students' levels of realizing the data about which information can be given to solve the problem if they are aware of missing information in verbal problems?
3. What are the 7th grade students' levels of recognizing excess information in verbal problems?

2. Method

2.1. Research Model

This study, which aims to examine 7th grade students' awareness of verbal problems containing missing information or excess information, is a descriptive study in terms of applying a data collection tool using a specific sample. The type of research that describes a specific situation qualitatively or quantitatively can be defined as descriptive research (McMillan and Schumacher 2001). In this study, the case study method was used because interviews were conducted and data collection tools were used to examine in-depth their awareness of problems with missing and excessive information. Case study methods are methods in which a single event or behavior is examined in depth, data are collected systematically and the real environment is examined (Subaşı and Okumuş 2017). Situations such as examining a specific situation in detail and selecting a limited sample were effective in the selection of this method.

2.2. Participants

While determining the participants of this study, the convenience sampling method was selected from purposive sampling methods. For the research, 15 students studying in the 7th grade in a public school in Şanlıurfa were interviewed. Since the processes of setting up and solving equations are intensively included in the mathematics teaching program in the 7th grade, this grade level was chosen as the sample. While selecting 15 students, attention was paid to the fact that they answered all the questions in order to enable them to make comments, to analyze their thoughts and to see where they were missing or where they made mistakes. Of the 15 students interviewed, 8 were female and 7 were male. The identities of the students were kept confidential and codes such as S1, S2 were used for the interviewed students.

2.3. Data Collection Tools and Collection of Data

While preparing the data collection tool, the literature was reviewed, middle school mathematics textbooks were examined and 6 open-ended questions were designed. Routine and non-routine verbal problems were taken into consideration in the selection of these problems. The problems were designed in such a way that 3 of them contained missing information and 3 of them contained too much information. Sub-directions were added under each problem in order to examine the students' questioning skills in detail. The application was carried out individually with each student and they were given enough time to solve the problems. The data collection tool was examined by 2 Turkish teachers and the meaningfulness and spelling rules in the questions were edited. After the data collection tool was prepared, it was solved by taking the opinions of 6 mathematics teachers and a field education expert and alternative answers to the problems containing missing information were tried to be determined. For example, the problem, "*The temperature decreases by an average of 0.5 °C for every 100 meters as you go higher than sea level. Ali, who measured the temperature of the water in his bag at sea level as 8°C, re-measured the temperature of the water when he reached the top of the mountain. Accordingly, what is Ali's result?*", was understood that the problem could not be solved by adding more than one piece of information and the problem was revised. Before the research was conducted, a pilot study was conducted with 20 volunteer students. In this pilot study, the appropriateness of the prepared questions to the grade level, the clarity of the questions, how the questions were understood by the students and the students' attitudes towards the questions were carefully examined.

Table 1. Problems used in the study

Type of problem	Problems
Problems with Missing Information	In one of the two classrooms in a school, the ratio of the number of girls to the number of boys is $\frac{3}{4}$. In the other class, the ratio of the number of girls to the number of boys is $\frac{2}{3}$. What is the ratio of the number of girls to the number of boys when the two classes are combined?
	Ayşe and her father made an agreement for Ayşe to study. According to this agreement, Ayşe will write down the correct and incorrect questions she solves during the week and tell her father at the end of the week. Her father will give her 1 ⁵⁰ liras for each question she solves correctly and take back 5 ² kurus for each question she solves incorrectly. Since Ayşe solved 10 ³ questions at the end of a week, how much money will she have?
	With the money in his pocket, Masal first goes to the stationery store and buys a notebook with 20% of his money. Then, with the remaining 40% of her money, she buys a movie ticket for the evening and keeps 25% of the remaining money, thinking that she can buy popcorn at the movie theater. In this case, how much money did Masal have at the beginning?
Problems with Excess Information	A microwave oven increases the temperature of food by 4 °C for every minute. It is turned on and decreases by 5 °C for every minute it is turned off. How many minutes should the microwave oven be switched on for the pastry at -6 °C to reach a temperature of 18 °C?
	A stationer sells pens for (x + 4) liras each for (2x - 3) liras each and notebooks for (4x + 5) liras each for (5x - 2) liras each. Write the profit of the stationer by selling 4 pens in algebraic notation.
	Faruk first spends $\frac{3}{10}$ of his money and then $\frac{5}{7}$ of the rest. Since he buys a chocolate bar from the market with 3 liras of the remaining 30 liras, how much is the total amount of his money?

The interviews conducted within the scope of the research were audio recorded with the consent of the parents. Before starting the interviews, the students were informed that no one other than the researcher would listen to the audio recordings. Before starting the interviews, the purpose of the interviews was explained to the students in order to prevent their excitement and to enable them to answer the questions more easily. In each interview, students were asked to make detailed explanations about the solution of the problems and to clearly express the thoughts that came to their minds during the solution. Each student who participated in the interview was asked about all the problems one by one and their thoughts were tried to be revealed.

2.4. Data Collection Tools and Collection of Data

In the data analysis, Gürsoy et al. (2015) and Gökkuurt Özdemir et al. (2018)'s theoretical frameworks for the analysis of problems with missing and excessive information were used and the data were analyzed descriptively. The categories and codes in the theoretical framework are given in Table 2.

Table 2. Categories and Codes

Categories	Sub-categories	Codes
Problems with Missing Information	Recognizing that there is missing information in the problem	Recognizing that there is missing information in the problem and solving the problem Recognizing that there is missing information in the problem and solving the problem but not being able to solve the problem
	Not realizing that there is missing information in the problem	Not realizing that there is missing information in the problem and not being able to solve the problem
Problems with Excess Information	Recognizing that there is excess information in the problem	Recognizing that there is excess information in the problem and being able to solve the problem Recognizing that there is excess information in the problem but not being able to solve the problem
	Not recognizing that there is excess information in the problem	Not Recognizing that there is excess information in the problem and not being able to solve the problem

While examining the students' awareness of problems with missing information, which information is necessary for the solution was examined in the interviews. For this purpose, the information required for solving problems with missing information was determined and tabulated in line with the opinions of mathematics teachers and a field expert. Table 3 shows the information that may be necessary for solving problems with missing information.

Table 3. Information that may be necessary for solving problems with missing information

Problems with Missing Information	Information Required for Solution
In one of the two classrooms in a school, the ratio of the number of girls to the number of boys is $\frac{3}{4}$. In the other class, the ratio of the number of girls to the number of boys is $\frac{2}{3}$. What is the ratio of the number of girls to the number of boys when the two classes are combined?	Number of students in both classes Number of girls in one class and number of boys in another class The ratio of girls in two classes to each other The ratio of males in two classes to each other The number of girls in one class is equal to the number of boys in another class or an algebraic relation
Ayşe and her father made an agreement for Ayşe to study. According to this agreement, Ayşe will write down the correct and incorrect questions she solves during the week and tell her father at the end of the week. Her father will give her 1^{50} liras for each question she solves correctly and take back 5^{22} kurus for each question she solves incorrectly. Since Ayşe solved 10^3 questions at the end of a week, how much money will she have?	Number of questions solved correctly Number of questions solved incorrectly The money Ayşe gave to her father Ratio of the number of correct to the number of incorrect An algebraic relation between the number of correct and the number of incorrect
With the money in his pocket, Masal first goes to the stationery store and buys a notebook with 20% of his money. Then, with the remaining 40% of her money, she buys a movie ticket for the evening and keeps 25% of the remaining money, thinking that she can buy popcorn at the movie theater. In this case, how much money did Masal have at the beginning?	Remaining money Notebook price Ticket price Corn price Algebraic relationship between the price of the book and the price of the ticket Algebraic relationship between the price of the notebook and the price of corn Algebraic relationship between the price of the ticket and the price of corn

2.5. Research Ethics

The authors declared that the research has been approved by Zonguldak Bülent Ecevit University Human Research Ethics Committee on 4 May 2023 with the protocol code 201.

3. Findings

3.1. Levels of Recognizing Missing Information in Verbal Problems

In this section, the answers given by the students in the interview for the problems containing incomplete information are analyzed according to the theoretical framework and given as a table.

3.1.1. Findings from the First Problem with Missing Information

As a result of the interview with the students regarding the first problem with missing information, the findings showing the awareness of missing information are given in the Table 4.

Table 4. Students' answers to the first problem with missing information

	Sub-categories	Codes	Students
In one of the two classrooms in a school, the ratio of the number of girls to the number of boys is $\frac{3}{4}$. In the other class, the ratio of the number of girls to the number of boys is $\frac{2}{3}$. What is the ratio of the number of girls to the number of boys when the two classes are combined?	Recognizing that there is missing information in the problem	Recognizing that there is missing information in the problem and solving the problem	
		Recognizing that there is missing information in the problem and solving the problem but not being able to solve the problem	S3, S7, S13
	Not realizing that there is missing information in the problem	Not realizing that there is missing information in the problem and not being able to solve the problem	S1, S2, S4, S5, S6, S8, S9, S10 S11, S12, S14, S15

According to the data in the Table 4, 3 students realized that there was missing information in the problem. They could not think that they should use different variables in different classes in the problem. When the answers of the students who gave incorrect answers are examined, half of them made rational addition and the rest added the ratios without equalization. When the student answers were analyzed, it was seen that most of them could not think that the ratios in the classes would be formed according to the class size. Some of them added the ratios directly, thinking that the given ratios represented the number of students, and some of them added the ratios with the denominator equalization method by making addition in rational numbers. This situation was thought to be caused by misconceptions in the rational numbers unit. During the interviews, it was seen that very few of the students realized the missing information, but those who realized it could not reach the solution. The fact that the first problem was different from the schema of the problems that the students had solved before made it difficult for them to understand the problem and realize that it contained missing information.

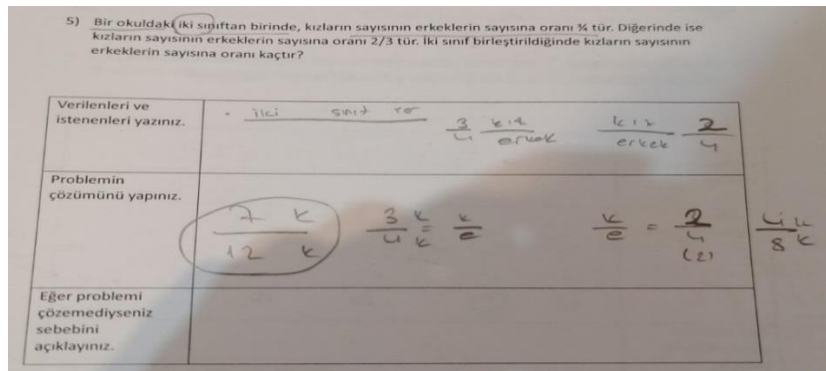


Figure 1. Student S13's response to the first problem

During the interview with the student S13, he was asked to explain his solution (seen in Figure 1). The student stated that he had to use variables for each class because the classes were different. However, he said that

he could not find out how to do the operations when given variables. With different questions asked during the interview, he could not realize that the question contained incomplete information. The fact that the student focused on adding the variables and went straight to the solution without interpreting what was given and what was asked made it difficult for him to realize the missing information.

S) Bir okuldaki iki sınıftan birinde, kızların sayısının erkeklerin sayısına oranı $\frac{3}{4}$ tür. Diğerinde ise kızların sayısının erkeklerin sayısına oranı $\frac{2}{3}$ tür. İki sınıf birleştirildiğinde kızların sayısının erkeklerin sayısına oranı kaçtır?

Verilenleri ve istenenleri yazınız. Çözüm	$\begin{array}{l} \text{A} \\ k_1 = \frac{3}{4} \\ \text{Erkek} = 4 \end{array}$	$\begin{array}{l} \text{B} \\ k_2 = \frac{2}{3} \\ \text{Erkek} = 3 \end{array}$	$\frac{3}{12} + \frac{8}{12} = \frac{11}{12}$
Problemin çözümünü yapınız. Veriler	$\begin{array}{l} \text{A} \\ k_1 = \frac{3}{4} \\ \text{E} = 4 \end{array}$	$\begin{array}{l} \text{B} \\ k_2 = \frac{2}{3} \\ \text{E} = 3 \end{array}$	$\rightarrow \frac{k_1 + k_2}{\text{Erkek}}$
Eğer problemi çözemediyse çözümünü			

Figure 2. Student S3's response to the first problem

The conversation with the student S3 during the interview is given below.

Researcher(R): Were you able to solve the problem?

S3: Yes.

R: Can you explain how you solved the problem?

S3: I added up what was given in the two schools.

R: I see that you added by equalizing the denominators. Who did you equalize when you equalized the denominators?

S3: Since the ones in the denominators are male students, I equalized the male students.

R: Well, is there such information in the question?

S3: No, it did not say equal, I think I chose the wrong solution.

R: Well, if you were to solve this question again, how would you solve it?

S3: I would solve it by giving values with x , but he didn't give the value of any of them.

R: What could you solve it with?

S3: I could solve it if he gave the number of girls or the number of boys.

The student S3 solved the question incorrectly at first, but with the questions asked by the researcher, he realized that the given data were insufficient to reach the correct solution. Although he wrote the data by dividing them into A class and B class, the fact that he equalized the number of boys showed that he could not interpret the data correctly. In order to reach the correct result with the questions asked, she realized that the question was incomplete, but she determined that she could reach the correct result when only the number of students was given. It was seen that when questions were asked to direct the student to think, it was easier for the student to realize that there was missing information in the problem.

The conversation with the student S11 during the interview is given below.

R: Did you solve the problem?

S11: Yes.

R: Can you explain how you solved the problem?

S11: I added girls with girls and boys with boys and found the new ratio.

R: So there were 2 girls and 3 boys in the first grade?

S11: No, he did not say that.

R: So could there be 4 girls and 6 boys in the first grade?

S11: Yes, we can expand the proportions. But then when I add them up, I would get a different result, I would have solved the question incorrectly.

R: So how would you solve the question if you were to solve it again now?

S11: I don't know, I cannot solve the question right now.

When the answer of the student S11 was examined, it was seen that he did not interpret what was given. The student was encouraged to think by asking questions about his/her solution and thus, he/she was directed to find his/her mistake by interpreting what was given. He realized that he solved the question incorrectly, but as a result, he still did not realize that the question contained incomplete information. It was observed that the missing information created confusion in the student.

Table 5. Students who recognize missing information in the first problem

Problems with Missing Information	Information Required for Solution	Students
In one of the two classrooms in a school, the ratio of the number of girls to the number of boys is $\frac{3}{4}$. In the other class, the ratio of the number of girls to the number of boys is $\frac{2}{3}$. What is the ratio of the number of girls to the number of boys when the two classes are combined?	Number of students in both classes	S3
	Number of girls in one class and number of boys in another class	S3, S7, S13
	The ratio of girls in two classes to each other	-
	The ratio of males in two classes to each other	-
	The number of girls in one class is equal to the number of boys in another class or an algebraic relation	S7, S13

According to the data in the Table 5, very few students were able to recognize the missing information. It was observed that none of the students interviewed realized that the solution of the problem could be reached by giving the ratio of girls in two classes or the ratio of boys in two classes. Students S3, S7 and S13 stated during the interview that the problem could be solved by giving the number of girls in one class and the number of boys in the other class. Students S7 and S13 stated that algebraic relations could be given, but they could not think that this could be done proportionally. From this point of view, it was seen that the students thought that more explicit information should be given in the questions.

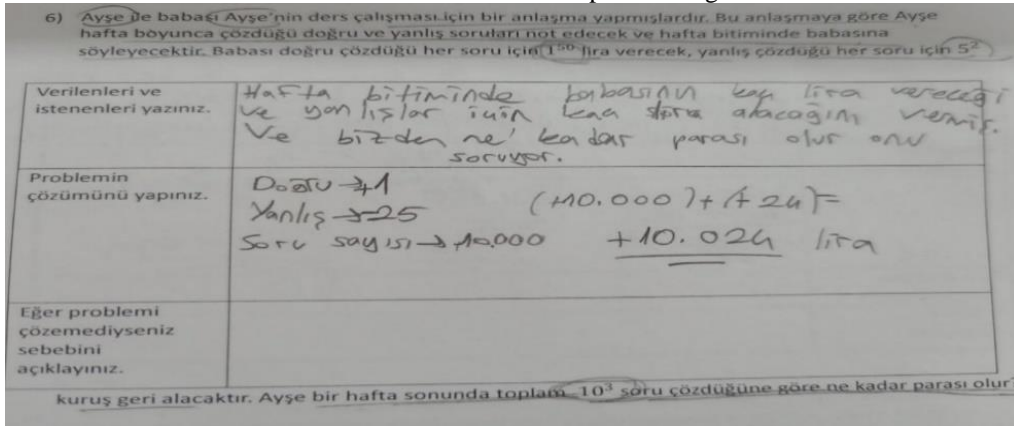
3.1.2. Findings from the Second Problem with Missing Information

The students' information on the subcategories and codes related to the second problem, which is one of the problems with missing information, is given in Table 6.

Table 6. Students' answers to the second problem with missing information

Sub-categories	Codes	Students
Ayşe and her father made an agreement for Ayşe to study. According to this agreement, Ayşe will write down the correct and incorrect questions she solves during the week and tell her father at the end of the week. Her father will give her 1^{50} liras for each question she solves correctly and take back 5^2 kuruş for each question she solves incorrectly. Since Ayşe solved 10^3 questions at the end of a week, how much money will she have?	Recognizing that there is missing information in the problem and solving the problem	S1, S2, S3, S4, S5, S6, S7, S8, S9, S11, S13, S14, S15
	Recognizing that there is missing information in the problem and solving the problem but not being able to solve the problem	S10
Not realizing that there is missing information in the problem	Not realizing that there is missing information in the problem and not being able to solve the problem	S12

In the second problem, it was observed that the majority of the students realized that there was missing information. The student S12 said that he made a mistake in the solution by thinking of the penny as lira and corrected this situation and went to the solution with the same operations again.

**Figure 3.** Student S2's response to the second problem

R: Did you solve the problem?

S11: Yes.

R: Can you explain how you solved the problem?

S12: I calculated the exponential expressions given. It costs 1 lira for correct answers, 25 kuruş for incorrect answers, and there are 1000 questions in total. When I subtract the right from the wrong, I get 24 liras. I added that to 1000.

R: Can you examine your solution again?

S12: 25 cents, 1 lira, I made a mistake there.

R: If you correct the kurus, will you have the correct solution? (The student read the question aloud again.)

S12: Yes, I will have solved it correctly.

Student S9 created an equation to solve the problem but left the equation halfway. When asked the reason for this, he said that he was interrupted because he did not know the number of correct values. However, he did not realize that he could reach the result with data other than the number of correct values. It was observed that the reason for this was that he focused on what was not given in the equation.

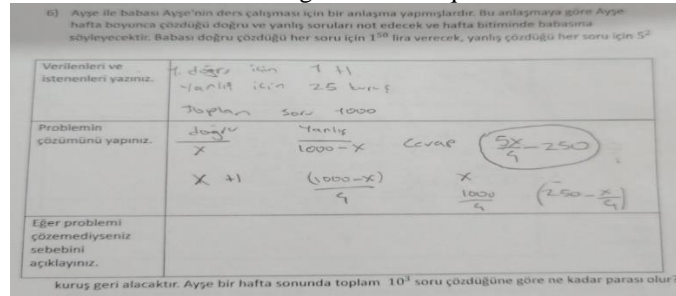


Figure 4. Student S9's response to the second problem

R: Did you solve the problem?

S9: No.

R: Can you explain how you tried to solve the problem?

S9: In the problem, I said x for the number of correct answers, there were 1000 questions in total, so there would be $1000-x$ number of incorrect answers. He gets 1 lira for correct answers and 25 kuruş for incorrect answers. Since 25 kuruş is one fourth of 1 lira, I divided it by 4. But then I couldn't continue.

R: Why couldn't you continue?

S9: Because I didn't know how many correct answers he got, so I couldn't write down the values. The question seemed wrong to me.

R: What would have helped you solve the question?

S9: In order to write x , I need to know the number of correct answers.

In the process of solving the problem, the student established the variables and formed algebraic expressions appropriate to the problem. He/she realized the missing information by thinking that he/she needed to establish an equation or equation to find the value of the variable in the problem. The student identified the information that may be necessary for the solution.

Table 7. Students who recognize missing information in the second problem

Problems with Missing Information	Information Required for Solution	Students
Ayşe and her father made an agreement for Ayşe to study. According to this agreement, Ayşe will write down the correct and incorrect questions she solves during the week and tell her father at the end of the week. Her father will give her 1 ⁵⁰ liras for each question she solves correctly and take back 5 ² kuruş for each question she solves incorrectly. Since Ayşe solved 10 ³ questions at the end of a week, how much money will she have?	Number of questions solved correctly	S1, S2, S3, S4, M5, S6, S7, S8, S9, S11, S13, S14, S15
	Number of questions solved incorrectly	S1, S2, S3, S4, M5, S6, S7, S8, S11, S13, S14, S15
	The money Ayşe gave to her father	S5
	Ratio of the number of correct to the number of incorrect	S7, S13
	An algebraic relation between the number of correct and the number of incorrect	-

According to the Table 7, except for the student S10 who was aware of the missing information but could not reach the correct solution and the student S12 who was not aware of the missing information, all of the students

who were aware of the missing information found that they could reach the solution of the problem when the number of questions they solved correctly or incorrectly was given. It was observed that none of the students interviewed realized that they could reach the solution by establishing an algebraic relationship between the number of correct and incorrect answers. The only student who stated that he could reach the solution by reaching the number of questions he solved correctly if he was given the amount of money he gave to his father was the student S5. Two students were able to find the correct answer by giving the ratio of the number of correct answers to the number of incorrect answers. Based on these data, it was observed that students thought of clear data more easily, but they did not think of complex data such as ratio and algebra. In the interviews, it was observed that this was due to the fact that students encountered questions with complex data less frequently. It is thought that the inactivity of the students in the problem posing step is also effective in this situation.

3.1.3. Findings from the Third Problem with Missing Information

The students' information on the subcategories and codes related to the third problem, which is one of the problems with missing information, is given in Table 8.

Table 8. Students' answers to the third problem with missing information

	Sub-categories	Codes	Students
With the money in his pocket, Masal first goes to the stationery store and buys a notebook with 20% of his money. Then, with the remaining 40% of her money, she buys a movie ticket for the evening and keeps 25% of the remaining money, thinking that she can buy popcorn at the movie theater. In this case, how much money did Masal have at the beginning?	Recognizing that there is missing information in the problem	Recognizing that there is missing information in the problem and solving the problem	S1, S2, S3, S6, S7, S8, S9, S11, S15
		Recognizing that there is missing information in the problem and solving the problem but not being able to solve the problem	S4, S10
	Not realizing that there is missing information in the problem	Not realizing that there is missing information in the problem and not being able to solve the problem	S5, S12

According to the Table 8, it was observed that the majority of the students were aware of the missing information. When the student answers were examined in the third problem, it was observed that some students who tried to solve the problem added the percentages and thought that the whole was 100, but they were not aware that they were asking about Masal's initial money. It was observed that some students started by saying that Masal's initial money was 100x, but they were interrupted. When asked why they stopped halfway, they gave answers such as "I couldn't find what to equalize from here on, I was confused, I think I started wrong" and they did not realize the deficiency. It was observed that students S4 and S10 could not reach the correct solution despite realizing the missing information. Student S4 stated that the reason he could not explain the deficiency was that he did not know the topic of percentages very well.

It was observed that students S5 and S12 were not aware of the missing information at all. Student S5 stated that he could not solve the problem because he could not establish proportion.

R: I see that you left the problem blank, but you didn't give a reason.

S12: Yes, I couldn't find a solution to the question.

R: Would you like to read the question again and solve it now? (The student reads the question out loud again.)

S12: He gave percentages but I can't figure out what to do.

According to the table 9, it was seen that none of the students could reach the data that the price of the notebook should be given in the form of ticket price, the price of the notebook should be given in the form of corn price, and the price of the ticket should be given in the form of corn price. Students S1, S2, S4, S9, S13, S15 found that the remaining money or the price of any of the products should be given. The majority of the students were able to reach the solution when the remaining money, the price of the notebook, the price of the ticket, and the price of the corn were given. However, they could not think that the prices could be given in kind.

It was noticed that the students could not think in detail about which data could be given in problems with missing information to reach the correct solution. They thought that they could reach the answers when more explicit data were given instead of proportional answers and expressions containing equations.

Table 9. Students who recognize missing information in the third problem

<i>Problems with Missing Information</i>	<i>Information Required for Solution</i>	<i>Students</i>
With the money in his pocket, Masal first goes to the stationery store and buys a notebook with 20% of his money. Then, with the remaining 40% of her money, she buys a movie ticket for the evening and keeps 25% of the remaining money, thinking that she can buy popcorn at the movie theater. In this case, how much money did Masal have at the beginning?	Remaining money	S1, S2, S3, S4, S6, S7, S9, S11, S13, S15
	Notebook price	S1, S2, S4, S6, S8, S9, S13, S14, S15
	Ticket price	S1, S2, S4, S6, S8, S9, S13, S15
	Price of corn	S1, S2, S4, S7, S8, S9, S13, S15
	Algebraic relationship between the price of the book and the price of the ticket	-
	Algebraic relationship between the price of the notebook and the price of corn	-
	Algebraic relationship between the price of the ticket and the price of corn	-

3.2. Findings Related to Students' Level of Recognizing Excess Information in Verbal Problems

In this section, the answers given by the students in the interview for the problems with excess information were analyzed according to the theoretical framework and whether the students realized which information was too much in the problems containing too much information was analyzed and given as a table.

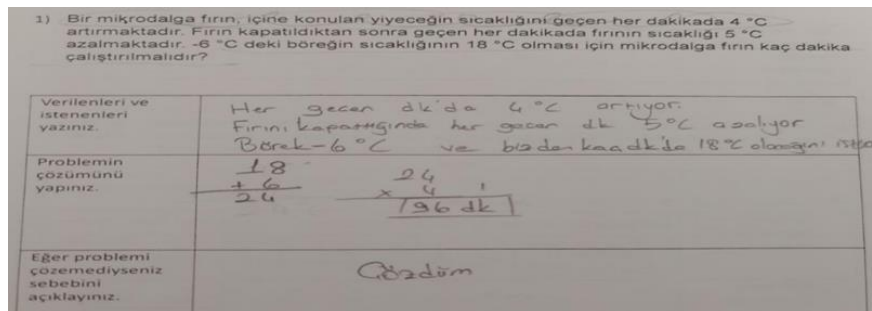
3.2.1. Findings from the First Problem with Excess Information

As a result of the interview with the students regarding the first problem with excess information, the findings showing the awareness of excess information are given in the Table 10 below.

Table 10. Students' Answers to the first problem containing excess information

	Sub-categories	Codes	Students
A microwave oven increases the temperature of food by 4 °C for every minute it is turned on and decreases by 5 °C for every minute it is turned off. How many minutes should the microwave oven be switched on for the pastry at -6 °C to reach a temperature of 18 °C?	Recognizing that there is excess information in the problem	Recognizing that there is excess information in the problem and being able to solve the problem	S1, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15
		Recognizing that there is excess information in the problem but not being able to solve the problem	S2
	Not recognizing that there is excess information in the problem	Not Recognizing that there is excess information in the problem and not being able to solve the problem	-

When the findings obtained from the answers to the problems with excess information were examined, it was seen that all of them recognized the excess information in the first problem, but only one student made an incorrect solution. The answer of the student S2 is given below. During the interview, he was asked to solve it again and he was able to reach the correct solution

**Figure 5.** Student S2's answer to the first problem

R: Why didn't you use some of the information?

S2: I didn't use it because the oven was never turned off

R: Can you explain your solution?

S2: I added 18 and 6. Then I multiplied it because it increases by 4 per minute, but I had to divide it.

3.2.2. Findings from the Second Problem with excess information

As a result of the interview with the students regarding the second problem with excess information, the findings showing the awareness of excess information are given in the table 11 below.

Table 11. Students' answers to the second problem with excess information

	Sub-categories	Codes	Students
A stationer sells pens for $(x + 4)$ liras each for $(2x - 3)$ liras each and notebooks for $(4x + 5)$ liras each for $(5x - 2)$ liras each. Write the profit of the stationer by selling 4 pens in algebraic notation.	Recognizing that there is excess information in the problem	Recognizing that there is excess information in the problem and being able to solve the problem	S1, S2, S3, S5, S6, S7, S8, S9, S10, S11, S12, S13, S15
		Recognizing that there is excess information in the problem but not being able to solve the problem	S4, S14
	Not recognizing that there is excess information in the problem	Not Recognizing that there is excess information in the problem and not being able to solve the problem	-

According to the Table 11, there were no students who did not realize that there was excess information, but two students made incorrect solutions. The solution of the student S4 is given below.

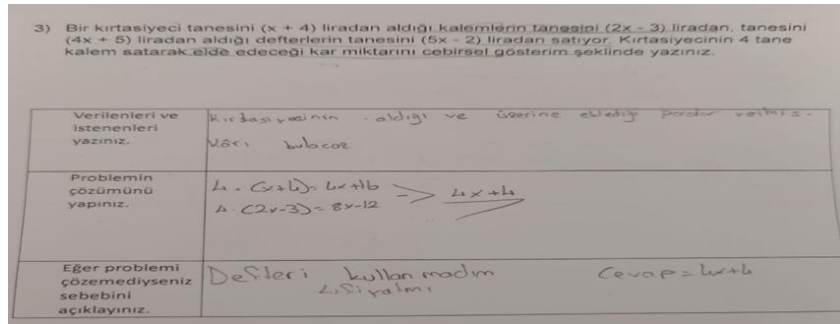


Figure 6. Student S4's answer to the third problem

Student S4 tried to do subtraction quickly and did not distribute the minus to the parenthesis and the result was incorrect.

3.2.3. Findings from the third problem with excess information

As a result of the interview with the students regarding the third problem with excess information, the findings showing the awareness of missing information are given in the table 12 below.

Table 12. Students' answers to the third problem with excess information

	Sub-categories	Codes	Students
Faruk first spends $3/10$ of his money and then $5/7$ of the rest. Since he buys a chocolate bar from the market with 3 liras of the remaining 30 liras, how much is the total amount of his money?	Recognizing that there is excess information in the problem	Recognizing that there is excess information in the problem and being able to solve the problem	S1, S3, S5, S6, S9, S10, S12, S13, S15
		Recognizing that there is excess information in the problem but not being able to solve the problem	S2, S4, S7, S11

Table 12 continued

	Not recognizing that there is excess information in the problem	Not recognizing that there is excess information in the problem and not being able to solve the problem	S8, S14
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According to the Table 12, the students S8 and S14 did not recognize the excess information, and the students S2, S4, S7, and S11 solved the question incorrectly although they recognized it. It was observed that the students who could not solve the question had a common deficiency in the subject of operations with rational numbers.

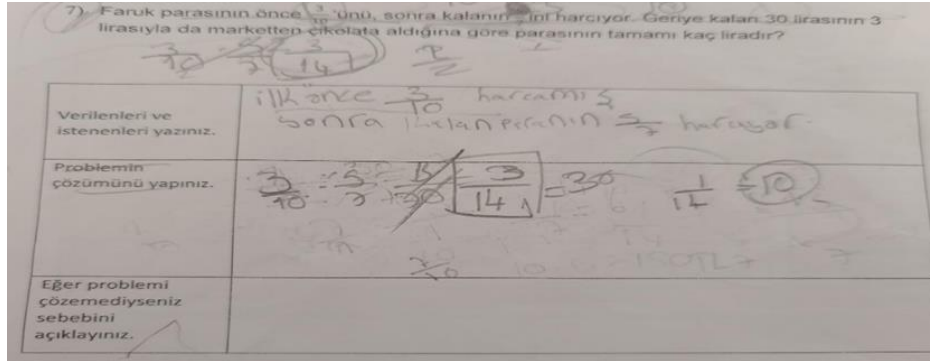


Figure 7. Student S14's answer to the third problem

In the interviews, the student S14 expressed his solution as follows.

R: Did you solve the problem?

S14: No.

R: Why do you think you couldn't solve the problem?

S14: I found 5/7 of 3/10, then I simplified it and thought it was equal to 30 liras, but I couldn't continue.

R: How did you think to do the rest?

S14: I was going to get the money from the equality and then I was going to add the chocolate bar he bought for 3 liras and find his original money.

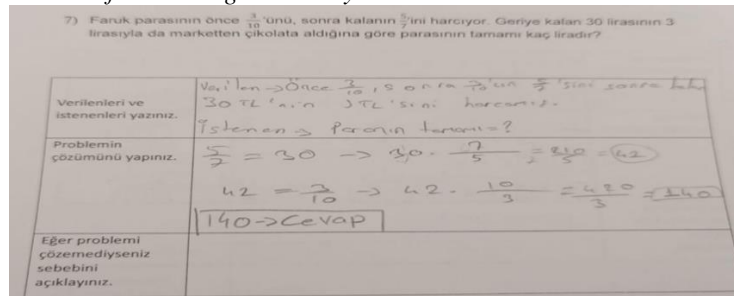


Figure 8. Student S7's answer to the third problem

In the interviews, The Student S7 expressed his solution as follows.

R: Did you solve the problem?

S7: Yes.

R: Can you explain your solution to me?

S7: If 5/7 of it is equal to the remaining 30 liras, it should be 42 liras. If 42 liras is equal to 3/10 of it, we have 140 liras.

R: Why didn't you use the 3 liras?

S7: He had already spent it out of 30 liras.

In his solution and explanation of the solution, the student determined that the excess information in the problem was 3 liras.

4. Discussion and Conclusion

In the study examining students' awareness of problems with missing and excess information, when the answers of 7th grade students to problems with missing information were analyzed, it was determined that half of them made mistakes and half of them left the question blank. When the answers of the students who left blank were examined, it was observed that the majority of them did not realize the missing information in the question and left it blank because they could not find what to do. From this point of view, it is concluded that students were insufficient in recognizing missing information in the problems they encounter, which supports the studies (Reusser and Stebler, 1997; Verschaffel et al. 1994; Gürsoy et al. 2015, Çevik and Gökkurt-Özdemir; 2018, Soylu and Aydın, 2006). The fact that the problems containing missing information were different from the schema of the problems that the students had solved before made it difficult for them to understand the problem and realize that it contained missing information. As Gürsoy et al. (2015) stated in their study, the fact that students solved result-oriented problems in the central exams and did not encounter different types of problems in the lessons caused them to have difficulty in identifying missing information. While solving problems with missing information, it came out that students had great difficulty in solving a problem and it was thought that the problem content affected the problem solution. Similarly, Öktem (2009) stated in his study that the problem content was effective, although not to a great extent, when students' realistic responses to problems were examined.

At the same time, similar to the previous studies, it was determined that the majority of 7th grade students solved problems with excess information without using excess information (Gürsoy et al. 2015; Çevik and Gökkurt-Özdemir 2018; Soylu and Aydın, 2006). When the data collection tool was analyzed, it was seen that even if they did not specify the excess information, they reached the solution of the problem without using it in the question. In the interviews, it was observed that they thought that the extra information was given to confuse them or to cause confusion. It was realized that some of the students who reached the wrong result did not use excess information but made mistakes due to processing errors or wrong solution methods. It was observed that the majority of the students who were not aware of the excess information focused on reaching the result by applying four operations on the question. It was observed that students were more successful in problems with more information compared to problems with missing information. In this context, Gürsoy et al. (2015) and Temiz and Çimen (2017) showed similarities with their studies. However, contrary to this study, Aydın Güç and Özmen (2012) stated in their study that students succeeded more easily in problems with missing information than in problems with redundant information and stated that the word "some" in the problem could be a clue.

It was observed that 7th grade students had difficulty in making comments while finding the data about which information could solve the problem if they were aware of the missing information in the verbal problems they encountered. It was observed that the students who found the data did not do comprehensive thinking and did not think about algebraic or proportional data. It was observed that instead of solving questions that would enable students to think comprehensively, they solved questions that they could directly reach the result with the given data, which prevented them from finding different data. It was observed that the majority of the students left the section "If you could not solve the problem, explain why" blank in the data collection tool. In the interviews with the students, their actual thoughts about this section were reached and they were allowed to comment on it. During the interviews, the students were encouraged to think about the problem with the questions directed to the students and thus they were provided with missing information. During the interviews, the students were asked which data could solve the problem when given in problems with missing information, and the students reached the data by interpreting the problem. Likewise, Naser (2008) stated in his study that conducting one-on-one interviews was useful in finding students' deficiencies in the problem solving phase and in seeing their mathematical thinking by examining their mistakes.

Similar to other studies, it was observed that students tended to solve the problem directly without determining the given and required information (Gökkurt Özdemir et al. 2018; Yenilmez and Yaşa 2007; Çetinkaya and Soybaş, 2018; Temiz and Çimen, 2017; Aydın and Özmen, 2012; Gürsoy et al. 2015; Çevik and Gökkurt Özdemir, 2018; Reusser and Stebler, 1997; Cook and Rieser, 2005). This situation made it difficult for them to recognize the missing and excess information in the question. The fact that students are generally raised in a way that focuses only on finding the correct answer without thinking about the problem and that they are not used to different types of problems may be effective on this situation. As Gökkurt Özdemir et al. (2018) also stated in their study, students had difficulty in analyzing problems. When student answers are analyzed, it was seen that the students focused on performing direct mathematical operations with the given numbers. During the implementation of the data collection tool, it was observed that the majority of the students tried to produce a solution to every question, even if it was absurd, in support of the concept of didactic contract. It was observed that they did not think about the problem, did not examine the data in the problem and focused on the result. Reusser and Stebler (1997) stated in their study that even when students were given unsolvable questions, they tried to solve them immediately. Even in questions with incomplete information, they tried to make operations

with the given information. In this study, the majority of the students did not think deeply and tried to reach a solution by making straight logic and doing four operations with the given data.

In order to increase students' awareness of problems with missing information and excessive information, they should encounter such questions in the lessons and make discussions on unsolved questions. Considering that the fact that students are more successful in problems that can be solved with existing knowledge is due to constantly solving questions with sufficient information, giving students problems with missing and excess information in the lessons will enable them to think comprehensively. In addition, students' awareness should be increased by including problems with missing and excess information in textbooks.

In the study, when asked which data were given in problems with missing information, most of the students gave the same data and could not think in detail. Considering this situation, along with problem solving activities, students should be supported with problem posing activities and it should be examined whether the problems they posed can be solved and discussed in the classroom. In addition, research can be conducted to examine students' learning, solutions and thinking processes in these learning environments.

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