Research Article

Investigating Mathematical Thinking through Productive Struggle: Mediated by Creative Learning and Moderated by Motivation

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Abstract: This study investigated the relationship between productive struggle and mathematical thinking, with a focus on the mediating role of creative learning and the moderating role of motivation. Productive struggle the process of grappling with challenging mathematical tasks was posited to enhance students' mathematical thinking, a crucial skill for academic success in mathematics. The research was grounded in the Anthropological Theory of Didactics and Behavioral Theories of Motivation, which provided a theoretical framework for understanding how students' engagement with challenging tasks could foster deeper cognitive development in mathematics. The study employed a descriptive survey research design anchored in the quantitative research approach, using Structural Equation Modeling (SEM) to analyze the data. A sample of Senior High School students was selected using a stratified random sampling technique to ensure representation across different demographic groups. Data were collected through validated questionnaires measuring productive struggle, creative learning, motivation, and mathematical thinking. The analysis, conducted using SEM, examined the direct effects of productive struggle on mathematical thinking, the mediating effect of creative learning, and the moderating effect of motivation. The results confirmed a significant positive relationship between productive struggle and mathematical thinking. Creative learning was found to partially mediate this relationship, while motivation had a negative but statistically insignificant moderating effect. Based on these findings, it was recommended that educators integrate tasks that promote productive struggle and creative problem-solving in mathematics curricula. Additionally, while motivation remains an essential factor in learning, further research was suggested to explore its role in different educational contexts.

Keywords: Creative learning, Motivation, Mathematical thinking, Productive struggle, Mediation analysis, Moderation analysis

1. Introduction

Mathematics plays an essential role in global education, especially in Ghana, where it is a cornerstone of the national curriculum and a key life skill. In an era defined by advancements in science and technology, mathematics is not only a tool for scientific inquiry but also a universal language that shapes societal progress and individual thinking (Aithal & Aithal, 2019). Numerous studies have underscored the relationship between mathematics and human behavior, suggesting that mathematical proficiency influences how individuals engage with the world (Karim et al., 2023). Despite the fundamental importance of mathematics, there is ongoing debate regarding the most effective pedagogical approaches to teaching and learning mathematics, both in Ghana and globally (Loveless, 2004).

A significant issue in mathematics education is students' conceptual understanding the ability to grasp underlying mathematical principles rather than merely memorizing procedures (Lithner, 2017). While strategies emphasizing conceptual understanding have gained popularity in recent years (Shield & Dole, 2013; Lithner, 2017), rote learning and repetition still dominate many classrooms, limiting students' deep mathematical reasoning (Lithner, 2017). Students often resort to recalling algorithms and applying learned methods, rather than exploring mathematical problems creatively, which hinders the development of true mathematical thinking (Jäder et al., 2019).

One approach that has gained attention is the use of productive struggle, where students engage with challenging problems that require perseverance and creativity. According to Livy et al. (2018), challenging problems often have multiple solutions and require various approaches to solve, fostering critical thinking. Productive struggle encourages students to attempt different strategies when initial solutions fail, promoting a deeper conceptual understanding (Perkins, 2016). The discomfort or confusion students experience during productive struggle, termed as mental unease (Pauli, 1960), has been shown to contribute to better problem-solving skills and conceptual learning (Roble, 2017).

In parallel, Creative Mathematical Reasoning (CMR) has been identified as a key component in fostering mathematical thinking. CMR encourages students to engage in higher-order thinking, using creativity to generate solutions to unfamiliar problems (Bieda & Staples, 2020; Lithner, 2017). This form of reasoning involves a shift

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from simply applying algorithms to developing new strategies, allowing students to solve novel problems effectively (Jonsson et al., 2014). Additionally, motivation plays a critical role in sustaining student engagement with challenging mathematical tasks. Students' motivation influences their willingness to persist through struggles and explore alternative solutions, which ultimately impacts their learning outcomes (Avsec & Sinigoj, 2016; Wu & Marsono, 2016).

This study seeks to enhance mathematical thinking through productive struggle, exploring how creative learning mediates this process and how motivation moderates it. By focusing on these interactions, the study aims to provide valuable insights into how mathematical reasoning can be fostered more effectively in the classroom, contributing to stronger mathematical competencies among students.

1.1. Problem Statement

While there has been considerable research on improving mathematical performance, much of it has focused on the relationship between math anxiety and performance (Beilock & Maloney, 2015; Brandenberger et al., 2018), rather than on the factors that specifically enhance mathematical thinking through productive struggle, creativity, and motivation. Although productive struggle has been shown to improve problem-solving skills (Hiebert & Grouws, 2007), limited research has explored the impact of motivation and creative learning on the process of developing mathematical thinking. The lack of focus on these factors highlights the need for more research into how motivation and creativity influence students' engagement with challenging mathematical problems.

Students' perceptions of their teachers' communication and instructional techniques significantly impact their engagement and success in mathematics (Gravemeijer et al., 2017). However, many teachers fail to recognize the importance of fostering cognitive creativity and often rely on over-scaffolding, which may limit students' opportunities to engage in productive struggle (Jonsson et al., 2014). Research suggests that teachers need further professional development to help them understand how to support students in overcoming mathematical challenges (Lukowski et al., 2019).

Although research on productive struggle has shown its benefits, there is a gap in understanding how creative learning and motivation influence this process. In particular, few studies have explored how these variables interact to enhance mathematical thinking and performance. Given the importance of productive struggle for developing deep mathematical understanding (Kapur, 2016), there is a need for more research on how it can be effectively mediated by creative learning and moderated by motivation. Furthermore, while various educational initiatives have aimed to improve mathematics education, their effectiveness has often been inconsistent, particularly in terms of how teachers perceive and address students' mathematical thinking.

This study will address these gaps by investigating the role of creative learning and motivation in enhancing mathematical thinking through productive struggle. By examining these mediating and moderating factors, the research aims to provide insights into how teachers can better support students in developing the skills necessary for solving complex mathematical problems and fostering creativity in their mathematical thinking. The findings will contribute to the body of knowledge and provide practical recommendations for teachers seeking to enhance their students' mathematical capabilities.

The study was guided by the following hypotheses:

H_1: Productive struggle has a direct positive effect on mathematical thinking**.**
H_2: Creative learning mediates the relationship between productive struggle and mathematical thinking.
H_3: Motivation moderates the relationship between productive struggle and mathematical thinking.

Figure 1 shows the conceptual framework of the study.

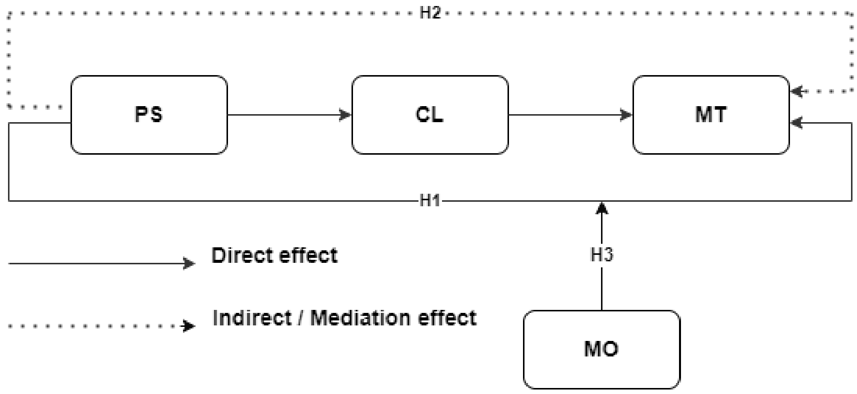

Figure 1. Conceptual Framework of the Study

Productive Struggle (PS) serves as the independent variable. By pushing students to think critically and experiment with different approaches, productive struggle promotes creativity in the learning process. As students apply concepts, reason, and solve complex problems, they strengthen their mathematical thinking abilities through the difficulties they face during productive effort. Creative Learning (CL) The creative learning is acting as a mediating variable between the relationship of productive struggle and mathematical thinking. Motivation (MO) Motivation is acting as a moderating variable on the relationship between productive struggle and mathematical thinking. Mathematical Thinking (MT) is acting as an independent variable in the research framework.

2. Literature Review

2.1. Theoretical Framework of the Study

This study was guided by the Anthropological theory of Didactics, Behavioral Theories of Motivation.

2.1.1. Anthropological Theory of Didactics

The Anthropological Theory of Didactics (ATD) centers on the concept of "praxeology," which studies human actions, such as learning mathematics, as forms of intentional conduct (Chevallard, 1992; Putra, 2016). According to Nicaud et al. (2004), mathematical actions, like simplifying rational algebraic expressions, can be understood through ATD as a comprehensive model for mathematical knowledge. ATD defines this knowledge through four essential elements: theory, technique, technology, and task type. These elements are organized into two primary models: the practical block, which includes the task (simplification of algebraic expressions) and the techniques (tools such as factorization and cancellation), and the knowledge block, which includes theory and technology that justify and support the techniques. Here, "technology" refers to the logical discourse explaining the "how" and "why" behind a technique (Nicaud et al., 2004). ATD views mathematical knowledge as evolving from human practices and is connected to traditional educational approaches, such as the German Didaktik Hopmann, 2007).

2.2.2. Behavioral Theories of Motivation

Behavioral theories, pioneered by scholars like Skinner (1953) and Bandura (1986), suggest that motivation in education is influenced by external rewards and punishments. Skinner's operant conditioning model posits that positive reinforcement encourages repeated behaviors, while negative reinforcement, through the avoidance of negative stimuli, also strengthens behavior. In education, rewards (grades, praise) promote student engagement (Skinner, 1953). Bandura's social learning theory expands this model by emphasizing observational learning, where students are motivated by observing peers' success (Bandura, 1986). Further advancements in behavioral theories, such as McClelland's model (1965, as cited in Covington, 1984), address the balance between students' need for success and their desire to avoid failure. Students are more likely to engage in tasks with a high probability of success, as their confidence in achieving goals plays a vital role in motivation. This is supported by research indicating that students who perceive success are more likely to persist through challenges (Dickinson & Butt, 1989). Behavioral theories underline the importance of building students' confidence and creating environments that reward effort, thereby fostering a growth mindset that encourages resilience and reduces the fear of failure.

2.2.3. Productive Struggle, Motivation and Mathematics Thinking

Cognitive engagement includes students' feelings of motivation, desire for self-directed learning, and perseverance (Abla & Fraumeni, 2019; Fung et al., 2018). Any time a student begins to solve a problem, the possibility of a struggle exists (Amidon et al., 2020); however, mathematics teachers need to intellectually challenge students in class, but need the support from other teachers and counselors for feelings of success and belonging (Fung et al., 2018). Persistence and use of creative problem-solving strategies are indicative of cognitive engagement (Fung et al., 2018). It is crucial that these ideals are maintained through student-teacher relationships to maintain focus on both content and connectedness (Abla & Fraumeni, 2019). These findings are consistent with the constructivism theory emphasizing that knowledge should not only be transferred; instead, it should be constructed by students through active participation during the learning process with the help of technology (Laz & Shafei, 2014). The mathematical creative thinking ability of the students was improved. There were groups that require more attention from the teacher even though the grouping was heterogeneous. This indicates that the level of students' mathematical creative thinking ability is varied. One of the reasons is that students are not yet accustomed to solving problems demanding higher order thinking and problem-solving skills. Mathematical creative thinking needs more time to create, develop, and enhance a work; it also requires a continuous practice of solving non-routine problems in the form of problem solving (Švecová et al., 2014). A study by Selvy et al. (2020) on students' mathematical creative thinking ability based on the pre-test and post-test results of each indicator presented shows there is an increase in students' mathematical creative thinking ability for each indicator in the experimental class and control class. Marashi and Khatami (2017) found that productive struggle greatly increases the creativity and motivate learners when the desirable outcome is accomplished. Learners' motivation to enhance their mathematical skills, promotes deep understanding of mathematical concepts, and supports the development of grit and creative problem-solving abilities.

To add up, motivation can start to develop, when kids probably need to feel comfortable with mathematics, thus be given challenges to overcome, and have high expectations for themselves (Middleton & Spanias, 1999). The results of these research indicate that the classroom atmosphere and a lack of supportive teachers may play a role in the drop in positive attitudes toward mathematics. These findings imply that motivational patterns are learned, and what is especially troubling is that students typically learn to dislike mathematics and that this dislike becomes an essential component of their mathematical self-concepts. These findings are consistent with results from national assessments (Dossey et al., 1988). The essence of mathematics is thinking creatively, not simply at the right answer," conferring to Ayele (2016). Collaborative problem solving which motivated by the team is essential for fostering mathematical creativity because it allows students to collaborate in small groups to accomplish a shared goal while sharing ideas and knowledge (Bae et al., 2013). Furthermore, according to Ayele (2016), "integration of the arts and student-led problem solving strategies can foster creative problem solving" (p. 3532).

Lepper et al. (1996) went so far as to propose that the judicious use of reward contingencies may help people gain enough expertise in an endeavor to foster the emergence of intrinsic motivation. Roble (2017) also contends that productive struggle in mathematics has a direct positive effect on mathematical thinking by enhancing students' understanding, motivation, and problem-solving skills. Research indicates that engaging in productive struggle leads to improved mathematics comprehension and sense making (ROBLE, 2017). If can construct it or create it, then they believe it more," said teacher Marie, meaning that when teachers allow students to discover properties and make observations on their own initially, they retain it better than if the teacher were to just tell them through lecture. Unfortunately, "kids want to be like robots," teacher Beth followed, as it requires less thinking and work on their part. In other words, they want to be able to follow a memorized process for every problem without having to figure out which way to solve it. As a result, we must objectively assess the creative potential of organizations and determine the circumstances in which they can achieve high levels of performance (Paulus, 2000). According to Woo et al (2009), group members' abilities as well as the volume and caliber of their interactions are key factors in cooperative learning. Therefore, in order to foster creativity within the group, appropriated team composition tactics are required.

Furthermore, productive struggle encourages students to explore multiple solutions and entry points, fostering a more flexible approach to learning mathematics. By experiencing and overcoming challenges during productive struggle, students not only deepen their understanding of mathematical concepts but also develop resilience and creativity in tackling complex problems, ultimately leading to improved mathematical thinking skills. Russo et al. (2020) also examined the views of teachers regarding the place of student struggle in the arithmetic classroom. Teachers who had built resilience revealed that studying mathematics required struggle. Mhakure and Sayster (2020) added that the constructive challenges that high school students had when simplifying logical algebraic formulas in a math lesson. Students' 9–12 struggle is important for student learning that tracend the ideal fruitful struggle and the most effective ways comprehensive understanding. Baker et al. (2020) found instances where a student studying fractions engaged in a productive struggle based on empirical data K–6 students. The study deduced that it can be challenging at first, encouraging the productive struggle is good for students' success as well as teachers' deeper understanding of their classrooms. However, Vazquez et al. (2020) argue that how parents view fruitful struggle and how to help their children learn math at home places the parental policies and programs ought to place a greater emphasis on particular actions and conduct that are backed by data, such as the productive struggle.

Students feel cognitively overloaded and confused about the task – this is evidenced by the fact that there are no written answers or attempts on paper. Students also claim that they do not remember the work and/or the type of problems, and there could be gestures of uncertainty, and resignation. As a sign of frustration, students' utterances could sound negative in term of simplifying rational algebraic expressions, students might not fully understand the illustration.

2.2.4. Productive Struggle, Creative Learning and Mathematics Thinking

In order to improve student performance in mathematics, it is imperative that students engage in productive struggle and creative learning. Further learning and comprehension in mathematics and other courses depend on productive struggle, which is defined by pupils taking on difficult assignments and making sense of complicated concepts (Biccard, 2024; Naza & Syamsuri, 2022).

In order to influence cognitive demand and learning outcomes, teachers are essential in providing opportunities for students to struggle productively and in properly addressing those problems (Naza & Syamsuri, 2022; Permanasari et al., 2022). Teachers can create an environment that encourages active student engagement, creativity, and invention in the learning process, which will boost academic outcomes, by combining both productive struggle and creative learning methods. In order to help students improve their mathematical creative thinking abilities and provide a variety of original answers to issues while they are studying, creative learning in mathematics is essential (Biccard, 2024). Research suggests that in the current competitive environment, mathematics education must foster creative thinking. It emphasizes the importance of developing students' creative abilities from an early age, especially in high school settings (Roth, 2019).

A number of research have looked into various methods for encouraging creative thinking in mathematics. One such method is the Aptitude Treatment Interaction model, which is based on mind mapping and has been demonstrated to successfully improve students' creative thinking abilities in mathematical contexts (Noris & Saputro, 2021). Furthermore, the kinds of studies carried out on students' cognitive styles and how they affect mathematical creativity emphasize how crucial it is to recognize individual variations in how one approaches and solves mathematical problems (Roth, 2019). All things considered, including creative learning strategies into mathematics instruction is essential for developing students' capacity for mathematical creativity and for equipping them with the skills they need to succeed in analytical and problem-solving activities.

In addition, Oksuz (2009) provided evidence that the capacity for mathematical thinking skills was required for successfully resolving mathematical problems. Mann (2005) also proposes that the focus of mathematics education should be on problem solving in order to give students the opportunity to work on complex mathematical tasks that call for divergent thinking and to maintain students' interest in the subject by appreciating and rewarding their mathematical creativity. However, secondary mathematics education programs did not effectively foster students' innovative thinking. The majority of math teachers did not receive much help or encouragement to develop fresh, efficient methods for enhancing their students' capacity for creative thought. Dell (2018) examined examples of the fruitful conflict between pupils in the fourth and fifth grades. Students with quality 7-8 Students in elementary schools have the opportunity to gain greater understandings of mathematics through positive effort.

Gray (2019) looked at students' productive struggle and how it affects their comprehension of mathematics. Based on his findings, Inservice Teachers creates assignments for overcoming problems that are in line with the fruitful fight. Ewing et al., (2019) also added Superior Preservice Teachers after the Second World War Establishing a link between students and mathematics subject, as well as giving them access to it. They concluded it can enhance attitudes and encourage pre-service teachers to engage their students in constructive conflict. Similarly, Murawska (2018) description of a task meant to encourage constructive conflict among preservice instructors. Preservice Teachers Activities that promote constructive struggle can support tenacity and the application of learning to real-world situations.

The impact of problem-solving instruction in order words productive or creative learning on students' achievement and problem-solving skills in the seventh grade was investigated in Pakistani government schools. The main finding was that the productive struggle teaching in seventh-grade mathematics proved more successful in raising students' mathematical achievement because, in the mathematics achievement test, students in the experimental group who received the new problem-solving instruction performed better than those in the group who received regular instruction. This conclusion is consistent with the research conducted by Ali et al. (2010), who found that using a productive struggle improved students' math achievement. One possible explanation for this could be that by using the productive struggle approach, students are able to apply fundamental concepts in both subject matter and required situations. The findings of Behlol et al. (2018) “The achievement level of high and low achievers students taught through PSA was significantly better than the performance of high and low achievers taught through traditional methods of teaching" are similar to the results that show that learners of all levels, i.e., low average and high achievers of the treatment group showed better achievements than the performance of participants treated by conventional teaching.

Furthermore, Cheng et al. (2018) findings that "Students' scientific knowledge, reasoning, and problem solving all are successfully improved after receiving six weeks scientific productive struggle". These are consistent with the study's finding that "students of the group treated with a new approach of students learning that s productive struggle did better than students of the group taught by routine teaching" in the ability test of productive struggle. Both average and low achievers benefited from it since it encouraged them to participate in classwork and utilize their minds to learn how to think, which improved their capacity for thought and the development of problem-solving skills. According to results from a recent study by Spooner (2022), modeling challenges students' boundaries by requiring them to do difficult independent work. Students get through the problem by having an open mind on how to find a solution. According to Dooley (2019), eliminating the challenge could reduce a task to procedures with no conceptual link. Therefore, Dooley argues that using cognitively challenging tasks is still essential for acquiring mathematical proficiency. The earlier study supports the findings of other researchers (Teo & Waugh, 2010; Runco & Johnson, 2002) and demonstrates that creative problem solving approaches enhance students' creativity. This study appears to support the idea that teachers who are conscious of and make an effort to nurture creativity in their pupils can do the same (Teo & Waugh, 2010). The improvement in students' problem-solving skills, however, was in line with the findings of Kopka (2010) and Hu et al. (2017), who asserted that problem-solving has laid the groundwork for an effective mathematics education. These findings appear to corroborate the findings of this study.

3. Methodology

3.1. Research Design

The descriptive design was adopted for the study. The descriptive design summarizes an existing phenomenon via the use of numbers to characterize individuals or groups and examines the nature of existing conditions (McMillan and Schumacher, 2010).

3.2. Participants and Sampling

The participants of the study included the form two form three schools in the Ashanti Region. The sample for the study consisted of 320 students in Form Two (2) out of the 1, 600 total students. The number of samples utilized in the study aligns with Yamane's (1967) recommendations for determining the optimal sample size for any descriptive design. Here is an illustration of this: $n = \frac{N}{1+Ne^2}$ where n is the sample size, N is the population in this case 1, 600, e is the error term that is confidence level at 95%

$$n = \frac{N}{1+Ne^2} = \frac{1{,}600}{1+(1{,}600)(0.05)^2} = \frac{1{,}600}{1+(1{,}600)(0.0025)} = \frac{1{,}600}{1+4} = \frac{1{,}600}{5} = 320$$

This study employed a combination of **stratified**, **proportionate**, and **simple random sampling**. First, students from each of the three selected schools were grouped into strata based on their academic programmes (stratified sampling). Then, a proportionate number of students was selected from each stratum based on its size (proportionate sampling). Finally, students within each stratum were randomly chosen (simple random sampling), ensuring that every student had an equal chance of being selected and that the overall sample was representative of the population.

3.3. Response Rate

Three hundred and twenty questionnaires as the sample of the study were given to the respondents and two hundred and sixty-one of them were returned and deem fit for the purposes of analysis. This means that the return rate of the questionnaires was 81% and this was considered good enough for analysis, because according to according to Mugenda and Mugenda (2003), a questionnaire rate of above 70% is excellent to represent the population of the study.

Table 1. Demographics of the Respondents

Demographics Characteristics	Frequency (N)	Percent (%)
Gender	**261**	**100.0**
Male	136	52.1
Female	125	47.9
Age	**261**	**100.0**
15 to 16 years	17	6.5
17 to 18 years	190	72.8
19 to 20 years	52	19.9
21 and above	2	.8
Programm	**261**	**100.0**
Science	85	32.6
Technical	59	22.6
General Arts	58	22.2
Business	59	22.6
Nameofschool	**261**	**100.0**
Prempeh College	98	37.5
Asanteman SHS	87	33.3
Agric Nzema SHS	76	29.1

The demographic data reveals that, out of the 261 respondents, regarding gender, 52.1% (136) of respondents were male, and 47.9% (125) were female, indicating a relatively balanced representation of both genders. In terms of age distribution, the majority of respondents, 72.8% (190), were between 17 and 18 years old, while 19.9% (52) were aged 19 to 20 years. Only 6.5% (17) were within the 15 to 16 years age range, and a small fraction, 0.8% (2), were aged 21 years and above. This distribution suggests that the majority of respondents fall within the typical age range for high school students. The program of study showed diverse interests, with 32.6% (85) enrolled in Science, making it the most popular program. The Technical and Business programs each attracted 22.6% of respondents (59), while 22.2% (58) were pursuing General Arts. This variety in academic programs reflects a balanced representation of academic fields within the sample. Finally, the schools represented in the study included Prempeh College, with the highest number of respondents at 37.5% (98), followed by Asanteman SHS at 33.3% (87), and Agric Nzema SHS at 29.1% (76). This distribution of respondents across the three schools provides a comprehensive view, ensuring that the findings reflect the perspectives of students from different institutions.

3.4. Data Collection Instrument

A structured questionnaire developed by the researcher was used to collect data from respondents. The instrument was organized into five sections (A–E). Section A focused on the demographic characteristics of participants, including gender, age, and program of study. Sections B to E were constructed in line with the study's objectives and measured the core constructs: productive struggle, creative learning, motivation, and mathematical thinking. Each construct was represented by ten (10) items, and responses were recorded using a five-point Likert scale, ranging from 1 (Strongly Disagree) to 5 (Strongly Agree). To ensure content validity, the instrument was subjected to thorough review by experts in mathematics education and educational research. Their feedback informed the refinement of items to ensure clarity and relevance. A pilot study was conducted with a subset of the target population to assess the reliability of the instrument. The results showed acceptable Cronbach's alpha coefficients (above 0.70) for all constructs, indicating strong internal consistency.

3.5. Data Analysis

The statistical software utilized for data analysis included Statistical Package for Social Sciences (vs. 23) and Analysis of Moment Structure (AMOS vr.23). In this study, Structural Equation Modeling (SEM) was employed to perform the inferential analysis. Zainudin (2015) suggests that SEM is an advanced model for conducting multivariate analysis.

4. Results

4.1. Exploratory Factor Analysis (EFA)

This section presents the results of the Exploratory Factor Analysis (EFA) conducted to assess the underlying structure of the measurement items (Table 2). The EFA was performed using Principal Component Analysis with Varimax rotation to identify the components representing Productive Struggle (PS), Motivation (MO), Creative Learning (CL), and Mathematical Thinking (MT).

Table 2. Exploratory Factor Analysis Results

Rotated Component Matrix				
Measurement Items	***Component*** 1	*2*	*3*	*4*
PS3				.846
PS5				.883
PS8				.838
PS9				.840
MO3		.945		
MO4		.935		
MO5		.940		
MO10		.934		
CL1			.885	
CL4			.862	
CL5			.872	
CL8			.880	
MT1	.890			
MT2	.926			
MT5	.786			
MT9	.818			
MT10	.895			

KMO and Bartlett's Test		
Total Variance Explained		81.444%
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.843
Bartlett's Test of Sphericity	Approx. Chi-Square	4007.259
	df	136
	Sig.	.000
Determinant		1.364E-7

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 5 iterations.

To explore the underlying structure of the measurement items related to Productive Struggle (PS), Motivation (MO), Creative Learning (CL), and Mathematical Thinking (MT), an Exploratory Factor Analysis (EFA) was conducted. The EFA was utilized using the Principal Component Analysis with Varimax rotation, a common method employed to clarify the factor structure by simplifying the interpretation of loadings (Akendita et al., 2024). The results, as displayed in Table 2, revealed four distinct components corresponding to the constructs under investigation. A minimum factor loading criterion of 0.50 was established. Acceptable levels of explanation were also ensured by evaluating the commonality of the scale, which shows the degree of variance in each dimension. Every commonality was over 0.50, according to the results. Measurement items with factor scores below 0.5 were eliminated since it was anticipated that a minimum of 0.5 would be obtained (Amoako et al., 2020). Using this approach, six measurement items were deleted under productive struggle, motivation and creative learning while 5 were deleted under mathematical thinking. The adequacy of the sample size for factor analysis was confirmed by the Kaiser-Meyer-Olkin (KMO) measure, which yielded a value of 0.843 which is far greater than the acceptable threshold of .5. According to Amoako et al. (2020) a KMO value above 0.8 is considered meritorious, indicating that the data were suitable for factor analysis. Moreover, Bartlett's Test of Sphericity produced a significant result with a Chi square value of 4007.259, degrees of freedom of 136, and a p value 0.001, further validating the appropriateness of factor analysis for this dataset. The analysis also showed that the four identified components together explained 81.444% of the total variance, a high percentage that underscores the robustness of the factor structure. Additionally, the determinant 1.364E-7 was acceptable for factor analysis (Akendita et al., 2024).

4.2. Confirmatory Factor Analysis (CFA)

Confirmatory Factor Analysis (CFA) was conducted to validate the factor structure identified through the Exploratory Factor Analysis (EFA) and to assess the fit of the measurement model (see Table 3).

Table 3. Confirmatory Factor Analysis

Model Fit indices: CMIN = 206.961; DF = 109; CMIN/DF = 1.899; TLI = .969; CFI = .975; GFI = .920; RMR = .021; RMSEA = .059; PClose = .116;		**Std. Factor Loadings**
PS	**PRODUCTIVE STRUGGLE (PS); CA = .897; CR=.898; AVE=.687**	
PS3	I understand that confusion and errors are part of learning mathematics	.829
PS5	I know that breaking through mathematics come from struggle	.871
PS8	I reflect on my struggles to identify what I can learn from them	.808
PS9	I will learn more if I am encouraging to stick with a mathematics problem even when I am not sure of how to solve it.	.809
MO	**MOTIVATION (MO); CA = .959; CR=.963; AVE=.898**	
MO3	I focus on my progress and improvement, rather than just outcome during mathematics lesson.	.966
MO4	Understanding the topics of mathematics is very important to me	.918
MO5	I think that learning mathematics is important because it stimulate my thinking	.927
MO10	I can do well in mathematics if my parents and teachers can give gifts.	.946
CL	**CREATIVE LEARNING (CL); CA = .870; CR=.871; AVE=.693**	
CL1	I am open to exploring new ideas and perspectives in my learning of mathematics	.824
CL4	I use my imagination to generate new ideas to solve mathematics problems	.828
CL5	My imagination and ingenuity distinguishes me from my friends in mathematics class.	.873
CL8	I am comfortable with ambiguity and uncertainty in the learning process of mathematics	.882
MT	**MATHEMATICAL THINKING (MT); CA = .967; CR=.967; AVE=.907**	
MT1	I try to see possible outcomes when I am solving mathematics	.886
MT2	When I encounter a difficulty in mathematics lesson, I first try to understand it.	.976
MT5	I use variety of strategies to solve mathematical problem.	.705
MT9	Problem solving is not an important part of mathematics	.754
MT10	Investigating why the solution to the mathematical problem is the right one is a waste of time	.893

The model fit indices are as follows: CMIN = 206.961, DF = 109, CMIN/DF = 1.899, TLI = .969, CFI = .975, GFI = .920, RMR = .021, RMSEA = .059 and PClose = .116 (Figure 2). These indices suggest that the model has an acceptable fit, indicating that the hypothesized model is consistent with the observed data (Shi & Maydeu-Olivares, 2020). The CMIN/DF (Chi-square/Degrees of Freedom Ratio) value of 1.899. Indicates a good fit, as values less than 3 are considered indicative of an acceptable model fit (Bayrak & Oğuz, 2021). Both the TLI (Tucker-Lewis Index) of .969 and the CFI (Comparative Fit Index) of .975 exceed the recommended threshold of .90, suggesting a good fit between the hypothesized model and the data (Schuberth et al., 2023). The GFI (Goodness of Fit Index) of .920, is above the preferred threshold of .90, suggesting a reasonable fit. The RMR (Root Mean Square Residual) value of .021 is below the .08 threshold, indicating a good model fit (Bayrak

& Oğuz, 2021). Additionally, the RMSEA (Root Mean Square Error of Approximation) value of .059, with a PClose value of .116, suggests that the model has a good fit, as it falls within the acceptable range (Schuberth et al., 2023). The standardized factor loadings for all the item, as presented in Table 3, demonstrate strong indicator reliability, with all loadings above the .50 threshold (Hair et al., 2010). The CFA results confirm that the measurement model is valid and reliable, with all constructs exhibiting strong internal consistency, convergent validity, and acceptable model fit Akendita et al., 2024). These results align with established thresholds in the literature, further reinforcing the robustness of the measurement model in this study (Fornell & Larcker, 1981; Hair et al., 2010).

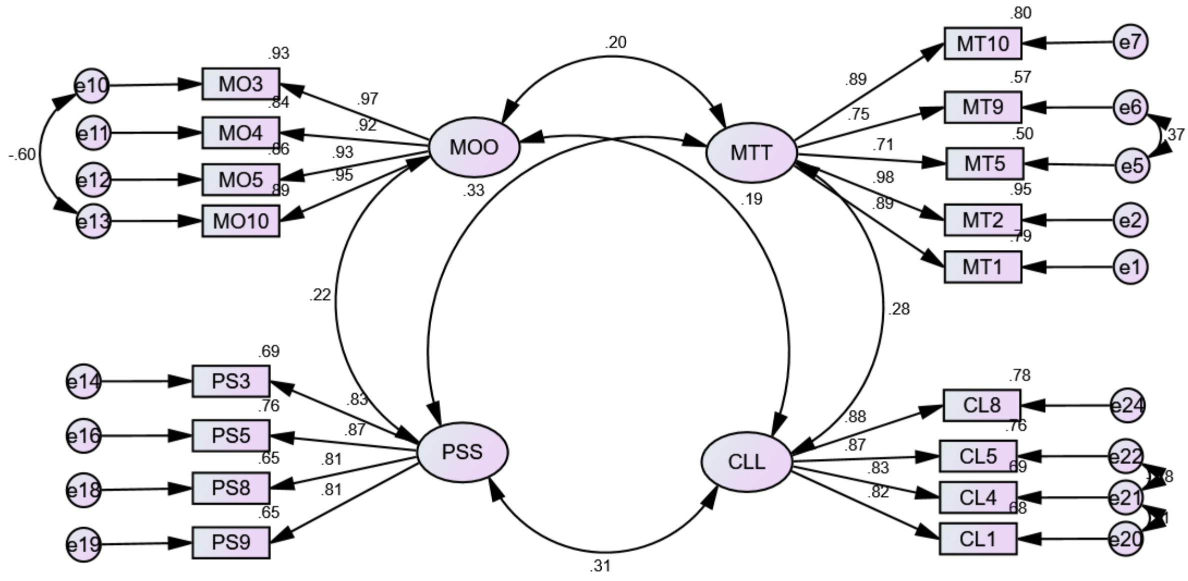

Figure 2. Confirmatory Factor Analysis

4.3. Descriptive Analysis and Discriminant validity

The descriptive statistics for the study variables, including Productive Struggle (PS), Motivation (MO), Creative Learning (CL), and Mathematical Thinking (MT), are summarized in Table 4. These statistics provide insights into the central tendencies and variations in students' perceptions and experiences related to these constructs. Discriminant validity is essential for ensuring that each construct measured in the study is distinct from the others. This validity is assessed by examining the extent to which each construct differs from others, ensuring that they measure unique aspects of the concept being studied (Schuberth et al., 2023). The results of the discriminant validity analysis are presented in Table 4.

Table 4. Descriptive and Discriminant validity Analysis

Variable	Mean	SD	CR	AVE	MT	MO	PS	CL
MT	4.14	0.74	0.927	0.720	**0.849**			
MO	4.31	0.57	0.968	0.882	0.198**	**0.939**		
PS	4.21	0.70	0.898	0.689	0.328***	0.216**	**0.830**	
CL	4.23	0.75	0.914	0.726	0.279***	0.191**	0.311***	**0.852**

*Note: * $p < 0.050$; ** $p < 0.010$; *** $p < 0.001$; $\sqrt{AVE}$ are bolded.*

Composite reliability (CR) and Convergent validity measures how closely each observed item interacts with the other observed variables on the same construct (Cheung et al., 2024). Expected values for the AVE and CR should be at least 0.7 and 0.5, respectively. In order to further examine the study and attain convergence validity, the AVE and CR were computed. The results demonstrate that the AVE and CR thresholds are met (Fornell & larcker, 1981). For the discriminant validity, the squares of the AVE values for each construct are compared with the individual inter-construct correlations, as presented in Table 4 (Fornell and Larcker, 1981). It emerged that all the squared AVE values were greater than each of the inter-construct correlations in other words the least of the squared root AVEs is greater than the largest correlation coefficient thereby satisfying the criteria for discriminant validity.

4.4. Path Analysis

The data was then subjected to additional analyses by looking at the potential relationship between the endogenous and exogenous variables of the framework used for the study, following the evaluation of the measurement model fit using Amos (v.23), Structural equation modelling (SEM). A mediation analysis was subsequently conducted using the direct and indirect effects based on bootstrap procedures (5,000 samples) and bias-corrected bootstrap confidence interval (95%). The study assessed the mediating role of creative learning (CL) on the relationship between productive (PS) struggle and mathematical thinking (MT). Furthermore, the moderating effect (interaction term) of motivation and the control variables were included into the analysis and are presented in Table 5 and graphically depicted in Figure 3. Overall, the model shows a good fit.

Table 5. Path Summary

Direct Effect	***Std. Estimates***	***S. E***	***C. R***	***P-Value***
Gender→MT	.098	.073	1.342	.081
Age→MT	-.011	.070	-.157	.850
Program→MT	.000	.032	.000	.996
School→MT	.051	.045	1.133	.367
PS→MT	.738	.317	2.323	.014
PS_MO→MT	-.122	.073	-1.663	.096
Indirect Effect	***Std. Estimate***	***Lower Bound***	***Upper Bound***	***P-Value***
PS → CL→MT	.051	.014	.106	.005

*Note: ***p = 0.1% significant value of p (.001)*

From the Table 5, the result showed that, the respondents' gender positively impacted MT and is statistically insignificant with 9.8% positive impact (*β = .098; CR = 1.342; p =.081*). MT was negatively impacted by age and is statistically insignificant with 1.1% negative impact (*β = -.011; CR = -.157; p=.850*). In addition, MT was positively impacted by the respondents' program of study and statistically insignificant with 0.0% positive impact on MT (*β = .000; CR = .000; p=.996*). Furthermore, from Table 5, the result showed that, the respondents' school positively impacted the MT but is statistically insignificant with 5.1% positive impact (β = .051; CR = 1.133; p =.367).

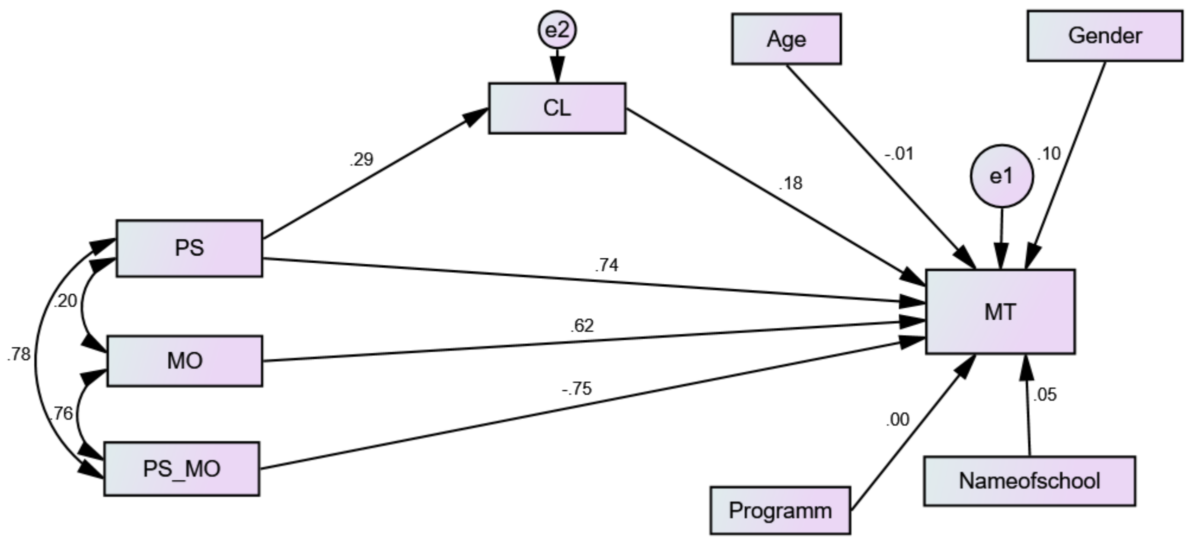

Figure 3. Path Analysis

4.4.1. Hypothesis One

H_1: Productive struggle has a direct positive effect on mathematical thinking.

Results on the hypothesized paths, as shown in Table 5 demonstrated how productive struggle (PS) has a positive effect on mathematical thinking and this impact was statistically significant, with 73.8% positive impact on mathematical thinking (β =.738; CR = 2.323; p =.014). That is, 73.8% increase in students' mathematical thinking is can be attributed to their productive struggle in the subject. Considering the study's findings, the null hypothesis, was consequently rejected and accepted the alternative hypothesis. Productive struggle has a direct positive effect on mathematical thinking.

4.4.2. Hypothesis Two

H_2: Creative learning mediates the relationship between productive struggle and mathematical thinking.

Using a 95% confidence level and a 5,000-bootstrap sample, the Bias-Corrected (BC) percentile method of bootstrapping was applied. The structural model shown in Table 5 satisfied all of the different fit indices as recommended by Hair et al (2010), just as the CFA did. Additionally, Figure 3 displays the study's diagrammatic representation of the structural model.

The hypothesis two which states that; Creative learning mediates the relationship between productive struggle and mathematical thinking was to determine whether Creative learning mediates the relationship between productive struggle and mathematical thinking.

As indicated in Table 5, results on the direct impact of productive struggle and mathematical thinking and demonstrated a statistically significant effect with 73.8 % positive impact on students' mathematical thinking (β =.738; CR = 2.323; p =.014). This implies that 73.8% improvement on students' mathematical thinking is attributed to their productive struggle. Similarly, the effect of creative learning on the relationship between productive struggle and mathematical thinking was statistically significant (β = .051; LB = .014; UB = .106; p = .005).

According to the study's findings, H2: Creative learning mediates the relationship between productive struggle and mathematical thinking was thus supported. This implies that Creative learning explains the relationship between the relationship between productive struggle and mathematical thinking. Since the direct effect of productive struggle is significant as well as the indirect effect, creative learning serves as a partial mediator between the relationship of productive struggle and mathematical thinking

4.4.3. Hypothesis Three

H_3: Motivation moderates the relationship between productive struggle and mathematical thinking.

Table 5 displays the moderating impact of motivation on the link between productive struggle (PS) and mathematical thinking (MT). The interaction term PS_MO as a moderator reports a direct negative effect on the relationship between productive struggle (PS) and mathematical thinking (MT). This was statistically insignificant with a *p*-value > 0.05 (β = -.122; C.R. = -1.663; *p* =.096). This indicates that the moderating variable (motivation) does not moderates the relationship between productive struggle (PS) and mathematical thinking (MT). Figure 3 also represent plots of two-way interaction effects (motivation).

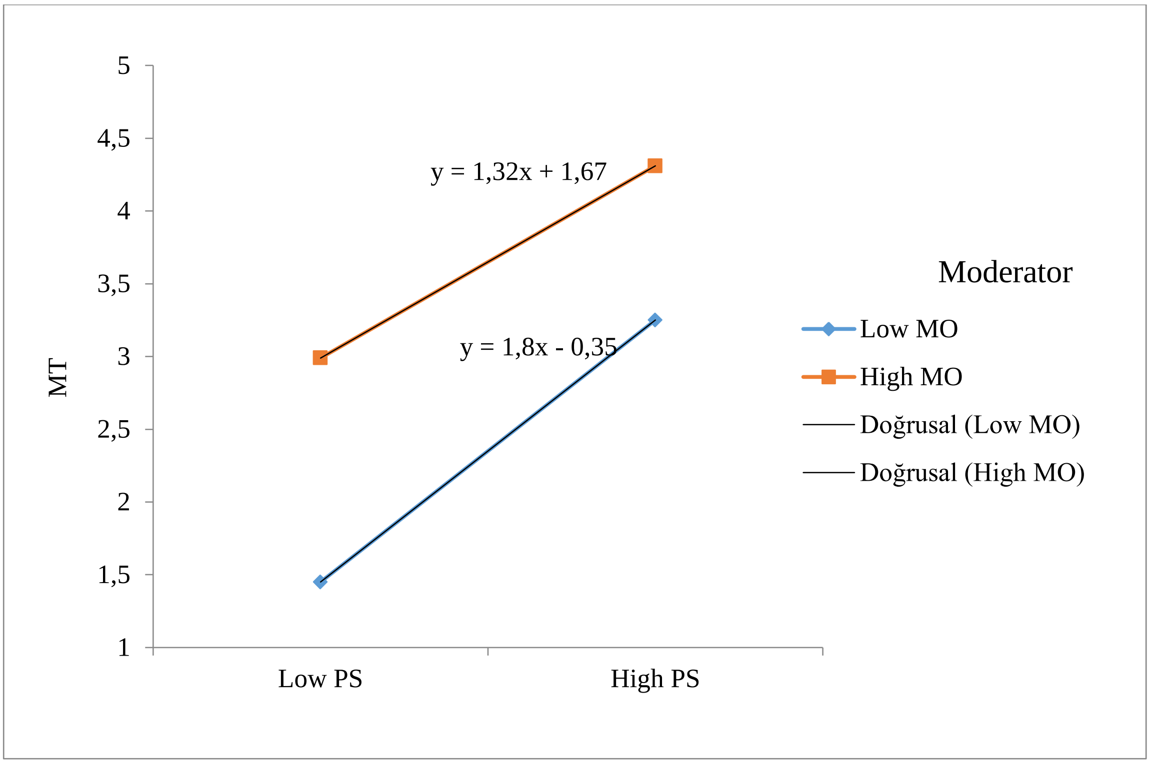

Figure 4. Plots two-way interaction effects

Figure 4 presents the plots of the two-way interaction effects of motivation. The graph indicates that motivation dampens the positive relationship between productive struggle and mathematical thinking, although this interaction is not statistically significant.

5. Discussion

The first hypothesis $\mathbf{H_{1:}}$ posited that productive struggle has a direct positive effect on mathematical thinking. The results supported this hypothesis, demonstrating that productive struggle significantly enhances students' mathematical thinking, with a 73.8% increase attributed to students' engagement in productive struggle. This finding aligns with the Anthropological Theory of Didactics, which emphasizes the role of didactic processes in shaping students' understanding and thinking in mathematics. According to this theory, productive struggle provides students with opportunities to engage deeply with mathematical concepts, facilitating the construction of knowledge through challenging yet achievable tasks (Chevallard, 1992; Putra, 2016). Empirically, these findings are consistent with prior studies that have highlighted the benefits of productive struggle in fostering mathematical understanding (Teo & Waugh, 2010). Productive struggle encourages students to grapple with problems, promoting a deeper comprehension of mathematical principles as they navigate through challenges. This process not only enhances problem-solving skills but also cultivates a mindset conducive to mathematical thinking.

The second hypothesis $\mathbf{H_{2:}}$ explored whether creative learning mediates the relationship between productive struggle and mathematical thinking. The results confirmed that creative learning serves as a partial mediator, significantly contributing to the relationship between productive struggle and mathematical thinking. This indicates that while productive struggle directly impacts mathematical thinking, creative learning enhances this relationship by providing students with innovative approaches to problem-solving. This finding can be understood through the lens of the Anthropological Theory of Didactics, which posits that learning environments and instructional strategies play a crucial role in students' cognitive development. Creative learning, in this context, acts as a didactic tool that facilitates the translation of productive struggle into meaningful mathematical understanding (Nicaud et al. 2004). It encourages students to approach problems from multiple perspectives, fostering flexibility in thinking and creativity in solution strategies.

Empirical studies have highlighted the significance of creative learning in mathematics education. For instance, a study by Hansen (2022) examined the interplay between students' agency, creative reasoning, and collaboration during mathematical problem-solving tasks. The findings revealed that when students engaged in collaborative problem-solving, they exercised greater agency and employed creative reasoning strategies, leading to enhanced engagement and deeper conceptual understanding. These results underscore the importance of integrating creative learning strategies in mathematics instruction to foster student engagement and conceptual comprehension.

The third hypothesis, $\mathbf{H_{3:}}$ proposed that motivation moderates the relationship between productive struggle and mathematical thinking. Contrary to expectations, the results indicated that motivation has a negative moderating effect on this relationship, although this effect was not statistically significant. This suggests that while motivation is generally considered an important factor in educational outcomes, it did not significantly alter the impact of productive struggle on mathematical thinking in this study. Behavioral Theories of Motivation, such as those proposed by Skinner (1953) and Bandura (1986), emphasize the role of motivation in shaping behavior and learning outcomes. These theories suggest that motivation drives students to engage in tasks, persist in the face of challenges, and ultimately achieve academic success. However, the findings of this study imply that motivation, in this context, may not have been a strong enough influence to moderate the effects of productive struggle on mathematical thinking. One possible explanation for this finding is that the nature of the tasks involved in productive struggle may have been intrinsically motivating for students, thereby diminishing the need for external motivation. This aligns with Deci and Ryan's (2000) Self-Determination Theory, which distinguishes between intrinsic and extrinsic motivation. When students are intrinsically motivated by the challenges of a task, the moderating effect of external motivation may be less pronounced. Empirical research on the role of motivation in mathematical thinking has produced mixed results. While some studies have found that motivation significantly enhances cognitive engagement and academic performance (Alba & Fraumeni, 2019; Fung et al., 2018), others have noted that the impact of motivation may vary depending on the context and nature of the tasks (Selvy et al., 2020). The findings of this study contribute to this nuanced understanding by suggesting that the role of motivation may be context-dependent, particularly in the realm of productive struggle and mathematical thinking.

6. Conclusion

The study concluded that productive struggle plays a critical role in enhancing mathematical thinking among students. The process of grappling with challenging tasks fosters deeper understanding and promotes cognitive development in mathematics. Creative learning served as an important mediator in this relationship, suggesting that when students engage in creative problem-solving, the benefits of productive struggle are amplified. However, the role of motivation as a moderator was not statistically significant, indicating that other factors may be more influential in shaping the relationship between productive struggle and mathematical thinking. This finding opens the door for further exploration into the complex interplay of factors that influence mathematical learning.

7. Implications

The findings carry meaningful scientific and practical implications for various educational stakeholders:

For Students: The results emphasize that learning through challenges helps build essential cognitive and problem-solving skills. Students should be encouraged to persevere through struggle, as it is an important and productive part of the learning process. This supports the development of a growth mindset in mathematics.

For Teachers: Educators should view struggle as a learning opportunity rather than a sign of failure. This calls for a shift in pedagogy away from quick answers and towards scaffolding that encourages exploration and creative thinking. Teachers also need support and training to implement such instructional methods.

For Educational Policymakers: The results suggest that curriculum frameworks should include opportunities for students to engage in productive struggle and creative learning. Policies supporting professional development in these areas would help improve mathematics instruction and student outcomes at scale.

8. Recommendations

Based on the findings of this study, several recommendations are proposed for educators, policymakers, and educational institutions:

- Integration of Productive Struggle in Curriculum Design: Educational curricula should be designed to incorporate tasks that challenge students and promote productive struggle. By doing so, students can develop resilience and perseverance, leading to enhanced mathematical thinking.
- Promotion of Creative Learning Approaches: Teachers should be trained in creative instructional strategies that encourage students to explore multiple solution paths and think outside the box. Creative learning should be emphasized as a means to bridge the gap between struggle and understanding.
- Motivational Strategies in Education: Although motivation did not significantly moderate the relationship in this study, it remains a critical factor in education. Schools should implement strategies to boost both intrinsic and extrinsic motivation, ensuring that students remain engaged and motivated to learn.
- Policy Development for Teacher Training: Policymakers should prioritize professional development programs that equip teachers with the skills to facilitate productive struggle and creative learning in the classroom. Such training will empower teachers to create learning environments that foster deep mathematical understanding.

9. Limitations

While the study yielded valuable insights, several limitations should be acknowledged:

Cross-sectional Design: The study employed a cross-sectional design, which limits the ability to make strong causal inferences. Longitudinal data would provide a clearer picture of how productive struggle and creative learning influence mathematical thinking over time.

Scope of Moderators and Mediators: The study focused only on motivation as a moderator and creative learning as a mediator. There may be other influential variables such as self-efficacy, peer interaction, or teacher support that could provide a more comprehensive model of mathematical thinking development.

Context-Specific Findings: The findings may be context-specific and may not generalize across different educational systems, levels, or cultures. Future studies should explore diverse settings and populations to enhance generalizability.

10. Future Research Direction

The study opens several avenues for future research:

- Exploration of Other Mediators and Moderators: Future research could explore other potential mediators and moderators that influence the relationship between productive struggle and mathematical thinking, such as self-efficacy, peer collaboration, or cognitive load.
- Longitudinal Studies on Motivation: Given the unexpected findings related to motivation, longitudinal studies could provide deeper insights into how motivation interacts with productive struggle over time and in different educational contexts.

- Application in Different Mathematical Domains: Further research could apply the study's framework to different areas of mathematics, such as geometry or algebra, to see if the relationships hold across various mathematical concepts.

Funding: The study received no funding support.

Ethics Declaration: All participants into the current study were given a consent form and asked to opt in whether they wanted to participate. No additional ethical approval was ¬required to conduct the study.

Data Availability: The corresponding author has access to the data supporting the findings of this study upon request.

Conflict of Interest: The author affirms that there were no conflicts of interest associated with the study.

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Appendix. Questionnaire for Students

Dear Student,
Your participation is kindly requested in completing this questionnaire. The information you provide will be treated with the utmost confidentiality, and your identity will not be disclosed. This questionnaire is strictly for academic purposes, and all data collected will be used solely for that purpose.
Thank you for your cooperation.

SECTION A (DEMOGRAPHIC DATA)

Kindly provide your information on the spaces below by ticking. (√)

1. **Age:** 11 – 15 years ( ) 16 – 20 years ( ) 21 years or above ( )
2. **Gender:** Male ( ) Female ( )
3. **What program are you pursuing?**
   Science ( ) Technical ( ) General Arts ( ) Business ( ) Others ( )
4. **Name of your School:**
   Prempeh College ( ) Asanteman SHS ( ) Agric Nzema SHS

SECTION B: PRODUCTIVE STRUGGLE
Please indicate your level of agreement or disagreement with the statement below. They have been rated in the form; **Strongly Disagree (SD), Disagree (D), Neutral (N), Agree (A), Strongly Agree (SA).**
Please Tick (√) in the box where appropriate.

CODE	Productive Struggle	SD	D	N	A	SA
PS1	I spend time through struggle with mathematical tasks without seeking for help					
PS2	I am able to bounce back from setback and failures to achieve my goal during mathematics lesson					
PS3	I understand that confusion and errors are part of learning mathematics					
PS4	I appreciate my efforts more than correct answers					
PS5	I know that breaking through mathematics come from struggle					
PS6	I ask questions that are related to my struggle and that help me to understand mathematics					
PS7	I do not give up easily in solving difficult problems					
PS8	I reflect on my struggles to identify what I can learn from them					
PS9	I will learn more if I am encouraging to stick with a mathematics problem even when I am not sure of how to solve it.					
PS10	I will learn more if I am allowed to make mistakes and receive delayed feedback					

SECTION C: CREATIVE LEARNING

Please indicate your level of agreement or disagreement with the statement below. They have been rated in the form; **Strongly Disagree (SD), Disagree (D), Neutral (N), Agree (A), Strongly Agree (SA).**

Please Tick (√) in the box where appropriate.

CODE	Creative Learning	SD	D	N	A	SA
CL1	I am open to exploring new ideas and perspectives in my learning of mathematics					
CL2	I am willing to take risks and try new approaches in my learning of mathematics					
CL3	I am encourage to think outside the box in my learning environment.					
CL4	I use my imagination to generate new ideas to solve mathematics problems					
CL5	My imagination and ingenuity distinguishes me from my friends in mathematics class.					
CL6	I generate a variety of ideas when faced with a problem or challenge during mathematics					
CL7	I take initiative to direct my own learning and explore topics of interest in mathematics					
CL8	I am comfortable with ambiguity and uncertainty in the learning process of mathematics					
CL9	I enjoy exploring different resources and materials to learn.					
CL10	I explore relationships between ideas and concepts that may not obviously be connected					

SECTION D: MOTIVATION

Please indicate your level of agreement or disagreement with the statement below.

They have been rated in the form; **Strongly Disagree (SD), Disagree (D), Neutral (N), Agree (A), Strongly Agree (SA).**

Please Tick (√) in the box where appropriate.

CODE	Motivation	SD	D	N	A	SA
MO1	I see learning of mathematics as an opportunity for personal growth and development					
MO2	I see mistakes as opportunities for when learning mathematics					
MO3	I focus on my progress and improvement, rather than just outcome during mathematics lesson.					
MO4	Understanding the topics of mathematics is very important to me					
MO5	I think that learning mathematics is important because it stimulate my thinking					
MO6	I want to know mathematics so that I can teach my friends and younger ones					
MO7	In mathematics class I prefer topics that really challenge me so I can learn new things					
MO8	I study hard in mathematics because I want to represent my school in mathematics competition					
MO9	I want do well in mathematics because it is important to show my ability to my family, friend, employer or others					
MO10	I can do well in mathematics if my parents and teachers can give gifts.					

SECTION E: MATHEMATICAL THINKING

Please indicate your level of agreement or disagreement with the statement below.

They have been rated in the form; **Strongly Disagree (SD), Disagree (D), Neutral (N), Agree (A), Strongly Agree (SA).**

Please Tick (√) in the box where appropriate.

CODE	Mathematical Thinking	SD	D	N	A	SA
MT1	I try to see possible outcomes when I am solving mathematics					
MT2	When I encounter a difficulty in mathematics lesson, I first try to understand it.					
MT3	I break down complex mathematical problems into manageable parts.					
MT4	I can think abstractly and understand complex mathematical concepts					
MT5	I use variety of strategies to solve mathematical problem.					
MT6	I pay close attention to detail when working on mathematical problems.					
MT7	I use mathematical knowledge in daily life					
MT8	I like to plan my mathematical lesson in advance					
MT9	Problem solving is not an important part of mathematics					
MT10	Investigating why the solution to the mathematical problem is the right one is a waste of time					