

Arithmetic and Algebra through Mathematical Structure: Performance and Solution Strategies of 6th- and 7th-Grade Students

Yaşar Akkan^a, Zeynep Natır^b, Betül Ergün^c and Asiye Nur Erdoğan^d

^aTrabzon University, Faculty of Fatih Education, Trabzon, Türkiye (ORCID: 0000-0001-5323-7106)

^bTrabzon University, Faculty of Fatih Education, Trabzon, Türkiye (ORCID: 0000-0002-6999-3835)

^cTrabzon University, Faculty of Fatih Education, Trabzon, Türkiye (ORCID: 0009-0003-7020-4834)

^dTrabzon University, Faculty of Fatih Education, Trabzon, Türkiye (ORCID: 0009-0002-1448-0722)

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Abstract: This study examined the performance and solution strategies of 6th- and 7th-grade students on structurally matched arithmetic and algebraic tasks within the framework of generalized arithmetic. An explanatory sequential mixed-methods design was employed with 150 students, including 90 sixth graders and 60 seventh graders, followed by task-based interviews with six students. Written responses were classified as correct, incorrect, or blank, while solution strategies were categorized as procedural/computational or relational/structural. The findings showed that correct-response rates were higher for arithmetic than for corresponding algebraic tasks at both grade levels and were generally higher among seventh-grade students. Procedural/computational strategies predominated, particularly in arithmetic tasks, whereas relational/structural strategies were observed more frequently in seventh-grade algebraic tasks. However, the use of a relational/structural strategy did not always result in a correct solution. Interview findings further illustrated that students' strategy use varied across tasks and mathematical structures. Overall, the findings indicate that performance and strategy use constitute related but distinct dimensions and that understanding the relationship between arithmetic and algebra requires attention not only to accuracy but also to how students recognize and use mathematical structure across different representations.

Keywords: algebraic thinking, generalized arithmetic, mathematical structure, solution strategies

1. Introduction

The relationship between arithmetic and algebra is one of the central topics in mathematics education research concerned with understanding the development of students' algebraic thinking. While arithmetic and algebra have traditionally been treated in school mathematics as largely sequential and separate domains of learning, contemporary approaches emphasize that these domains are closely related through shared mathematical structures, including number, operation, equivalence, relationships, and generalization (Cai & Knuth, 2005; Radford, 2018; Subramaniam & Banerjee, 2011). From this perspective, algebraic thinking is regarded not merely as the ability to operate with letters and symbols, but as a process of recognizing mathematical relationships, expressing these relationships through different representations, generalizing, and reasoning on the basis of these structures (Kaput, 2008; Kieran, 2018, 2022). Accordingly, examining the development of students' algebraic thinking requires attention to how mathematical structures are recognized and used before and as students begin to work with algebraic symbols.

In this context, generalized arithmetic, one of the key components of the early algebra approach, provides an important framework for understanding the conceptual continuity between arithmetic and algebra. Generalized arithmetic involves moving beyond activities carried out with particular numbers and operations to consider the properties of numbers and operations, equivalence relations, and regularities among them as more general mathematical relationships (Kaput, 2008; Warren, 2003). Accordingly, relationships such as the commutative, associative, and distributive properties, inverse operations, compensation, and equivalence are viewed not merely as properties that support the execution of arithmetic operations, but also as resources through which students can recognize more general mathematical structures and reason on the basis of those structures (Carpenter, Levi, Franke, & Zeringue, 2005; Kieran, 2022). Thus, arithmetic is not regarded simply as a procedural domain that must be mastered before algebraic thinking can develop, but as a context in which students can recognize and reason about mathematical relationships and structures.

The recognition of mathematical structures and the use of those structures during problem solving are particularly important for understanding the relationship between arithmetic and algebra. In this study, structure sense is associated with forms of structural thinking that become visible when students recognize relationships among the components of mathematical expressions and use those relationships in their solution processes (Kieran, 1988; Linchevski & Livneh, 1999; Subramaniam & Banerjee, 2011). Accordingly, decomposing and

Corresponding Author: Yaşar Akkan  email: yasarakkan@trabzon.edu.tr

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recombining numbers and operations in different ways, making use of the properties of operations, recognizing equivalent expressions, and restructuring mathematical expressions in accordance with the purpose of a solution are considered indicators of structural thinking (Carpenter et al., 2005; Mason, Stephens, & Watson, 2009). For example, in an equality of the form $78 + \square = 81 + 24$, a student may first calculate the right-hand side and then determine the unknown. Alternatively, the student may recognize that 81 is three more than 78 and use this relationship to balance the equality. Although both approaches lead to the same result, they differ in how the mathematical structure is used during the solution process. In the latter approach, the student uses the relationship between the expressions as the basis for the solution rather than simply carrying out the operations (Jones, Inglis, Gilmore, & Dowens, 2012; Kieran, 2022).

Nevertheless, the transfer of mathematical structures recognized in arithmetic contexts to algebraic situations is not an automatic process. Research shows that although students may recognize mathematical relationships in tasks involving particular numbers, they may experience difficulty using those same relationships when they are represented through letters and symbols (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994; Subramaniam & Banerjee, 2011). The relationship between arithmetic and algebra has therefore been argued to extend beyond a unidirectional progression from arithmetic to algebra; algebraic generalization may also contribute to students' reconceptualization of arithmetic relationships as more general mathematical structures (Britt & Irwin, 2011; Mason, 2018). This perspective suggests that the relationship between arithmetic and algebra should be examined not only in terms of the transfer of students' arithmetic knowledge to algebra, but also in terms of how students engage with structurally related mathematical relationships when these are presented in different representations and contexts.

From this perspective, determining merely whether students obtain correct answers is insufficient. The same mathematical result can be reached through different solution approaches, and these approaches can provide different insights into how students engage with mathematical structure. A student who carries out operations step by step, substitutes particular values, or calculates and compares results may be using a procedural/computational approach, whereas a student who draws on the properties of numbers and operations, equivalence, compensation, or the common structure of expressions may be demonstrating a relational/structural approach (Carpenter et al., 2005; Linchevski & Livneh, 1999; Kieran, 2022). However, the use of a procedural strategy does not, in itself, indicate that a student has failed to recognize mathematical structure, just as the use of a structural strategy does not guarantee a correct solution in every case. Therefore, examining performance and solution strategy as distinct dimensions is important for revealing both how successfully students solve tasks and how they use mathematical structures in the course of solving them.

Within this framework, continuity between arithmetic and algebra needs to be examined not only by comparing students' performance across the two types of tasks, but also by examining whether solution strategies associated with the same mathematical structure are maintained across different representations. In other words, an important question is whether a relationship that a student recognizes and uses in an arithmetic context also serves as the basis for solving an algebraic task in which the same structure is presented in symbolic form. Arithmetic and algebraic tasks matched in terms of mathematical structure provide an opportunity to examine this continuity more directly. Such a comparison makes it possible to determine whether students maintain the solution approaches they use in arithmetic when working with algebraic tasks involving similar mathematical structures, and to examine the relationship between arithmetic and algebra through continuities and changes in students' solution strategies.

In this study, rather than addressing all dimensions of early algebraic thinking, the focus is placed on the use of mathematical structures that can be examined within the framework of generalized arithmetic. Accordingly, the properties of numbers and operations, together with the relational meaning of the equal sign and equivalence, constitute the main domains of mathematical structure addressed in the study. Structure sense is not treated as an independent psychometric construct to be measured; rather, it is conceptualized as a characteristic of thinking that becomes visible when students make mathematical relationships the basis of their solution processes while solving particular tasks. Thus, the primary unit of analysis is the solution approach that students demonstrate in specific arithmetic and algebraic tasks. Consistent with this approach, performance and strategy type are examined as separate dimensions. In addition to whether students obtain correct or incorrect results, the study considers the extent to which, and the ways in which, students use mathematical structure in reaching their solutions.

The 6th and 7th grade levels provide a particularly relevant context for examining the transition between students' arithmetic experiences and their increasingly systematic engagement with variables and algebraic expressions. At these grade levels, students continue to draw on their prior experiences with the properties of numbers and operations, as well as with equality and equivalence, while also beginning to work with these relationships in symbolic representations. Examining students' performance and solution strategies in arithmetic and algebraic tasks matched in terms of mathematical structure can therefore contribute to a more detailed understanding of the continuities and differences that emerge between arithmetic and algebra. Accordingly, this

study compares students' performance in arithmetic and algebraic tasks and the procedural/computational and relational/structural strategies they employ in order to examine how structurally matched mathematical relationships are engaged with across different forms of representation.

Purpose of the Study

Taken together, the relevant theoretical and empirical literature suggests that the relationship between arithmetic and algebra cannot be explained solely in terms of the extent to which arithmetic performance predicts subsequent algebra performance. A central issue in understanding this relationship is whether students maintain and use the mathematical relationships they recognize in arithmetic when they encounter similar structures in algebraic representations and contexts. Subramaniam and Banerjee (2011) note that the spontaneous transfer of structure sense developed in arithmetic contexts to algebraic situations cannot be assumed, while Kieran (2022) emphasizes the need to examine more closely the relationship between the structural dimensions of arithmetic and algebraic thinking. Accordingly, evaluating the relationship between arithmetic and algebra solely on the basis of correct responses provides limited information about how students recognize, represent, and use mathematical structures. There is therefore a need to examine not only students' performance in structurally matched arithmetic and algebraic tasks, but also the strategies they employ during the solution process and the extent to which these strategies draw on the mathematical structure embedded in the tasks.

The purpose of this study is to examine the performance and solution strategies of 6th and 7th grade students in arithmetic and algebraic tasks matched in terms of mathematical structure. Specifically, the study examines how students' performance and strategy use vary according to task type and grade level and seeks to identify, through an analysis of students' explanations of their solution processes, the forms of reasoning reflected in the strategies they employ. In this way, the study addresses not only whether students obtain correct results, but also how they engage with structurally matched mathematical relationships when these are presented in arithmetic and algebraic representations and how they draw on procedural/computational and relational/structural approaches during the solution process. In line with this purpose, the study addresses the following research questions:

1. How does the performance of 6th and 7th grade students in arithmetic and algebraic tasks vary according to task type and grade level?
2. How do the strategies used by 6th and 7th grade students in solving arithmetic and algebraic tasks vary according to task type and grade level, and how do students explain the strategies they use?

2. Method

2.1. Research Design

In this study, a mixed-methods approach was adopted to holistically examine the performance levels of 6th and 7th grade students on arithmetic and algebraic tasks, as well as the solution strategies they employed. The research was conducted in accordance with the explanatory sequential design, one of the mixed-methods research designs. In the explanatory sequential design, the research process begins with the collection and analysis of quantitative data, followed by the collection and analysis of qualitative data in order to further explain the quantitative findings (Creswell & Plano Clark, 2018). In the quantitative phase of the study, students' performance levels were determined through an analysis of their written responses to arithmetic and algebraic tasks. Subsequently, students' solution processes were analyzed in terms of procedural/computational and relational/structural strategies. In this way, the distribution of performance levels and the solution strategies employed was revealed according to task type and grade level.

In the qualitative phase, task-based interviews were conducted in order to examine in greater depth how the distributions that emerged in the quantitative findings were reflected in students' solution processes. In the interviews, students were asked to explain the solution methods they employed in arithmetic and algebraic tasks, and thus it was examined how the performance levels identified through written responses, together with the procedural/computational and relational/structural strategies, were reflected in students' thinking processes. Accordingly, in this study, quantitative and qualitative data were used sequentially and in a complementary manner. The general performance and strategy findings identified in the quantitative phase were explained and further deepened through the student explanations and solution processes obtained in the qualitative phase. In this way, the two main areas of findings in the study—performance level and solution strategies—were examined jointly, both in terms of group-level distributions and students' individual solution processes (Creswell & Plano Clark, 2018).

2.2. Participants

The study group of the research consisted of a total of 150 students enrolled in three secondary schools located in the central district of a province in the Eastern Black Sea Region. Of these students, 90 were at the 6th

grade level and 60 were at the 7th grade level. In determining the quantitative study group, access to the research setting, the feasibility of implementation, and the availability of voluntary participation were taken into consideration, and accordingly, the convenience sampling method was employed. Convenience sampling is one of the non-probability sampling methods based on the inclusion of participants who are accessible to the researcher and appropriate for the purpose of the research (Etikan, Musa, & Alkassim, 2016). In the qualitative phase of the study, criterion sampling, within the scope of the purposive sampling approach, was utilized in order to examine in more detail the different performance and solution strategy tendencies that emerged as a result of the quantitative implementation. Purposive sampling enables the selection of participants who can provide detailed information about the phenomenon under investigation, in line with specific characteristics and criteria (Palinkas et al., 2015). Accordingly, taking the results of the quantitative implementation into consideration, six students—three at the 6th grade level and three at the 7th grade level—were determined for the interviews. In selecting the students, the criteria of voluntary participation, the ability to express their thinking verbally, variation in task performance and solution-strategy tendencies observed in the written data, and teacher recommendation were taken into consideration. This variation was evaluated based on the distribution of students' correct, incorrect, and unanswered responses to the written tasks, together with the types of strategies they employed in their solutions. In this regard, attention was paid to ensuring that the interview group included students who predominantly used procedural/computational strategies, those who provided relational/structural explanations, and those who used both types of strategies across different tasks. Thus, the interview participants were selected to reflect diversity in terms of task performance and solution-strategy tendencies.

The interview sample was therefore selected to provide explanatory variation in individual solution processes rather than to reproduce the quantitative distribution of the full sample. Because verbal expressiveness and teacher recommendation were among the selection criteria, the interview findings are interpreted as explanatory qualitative evidence rather than as representative estimates. In order to protect the confidentiality of the students who participated in the interviews, the pseudonyms Deniz, Miray, Aleyana, Zeynep, Bahar, and Ali were used instead of their real names. In addition, the necessary permissions were obtained from the school administrations in order for the implementations to be carried out at the schools where the research was conducted. In determining the students who would participate in the task-based interviews in the qualitative phase, the principle of voluntariness was taken as a basis, and students' participation in the research was carried out with the knowledge and consent of their families. The preference for the 6th and 7th grade levels in the study was influenced by the fact that these grades represent a period in which students' transition from arithmetic thinking to algebraic thinking, in which letters and symbols are used, becomes evident. At these grade levels, students have a certain degree of arithmetic experience regarding the properties of numbers and operations as well as equality and equivalence, while at the same time they begin to encounter variables and algebraic expressions in a more systematic manner. For this reason, the 6th and 7th grades constitute an appropriate transitional period for examining how students engage with structurally related mathematical relationships across arithmetic and algebraic representations (Kieran, 2022; Subramaniam & Banerjee, 2011).

2.3. Data Collection Instruments

The data collection process in the study began with the administration of a task set consisting of 10 tasks prepared for the purpose of determining students' performance levels on arithmetic and algebraic tasks and the solution strategies they employed. Subsequently, task-based interviews were conducted using the same task set in order to examine students' solution processes and the strategies they employed in greater detail. The tasks were developed for this study on the basis of the generalized arithmetic framework and the three mathematical domains specified below. Expert review was sought to evaluate the appropriateness of the task set for the target grade levels, the mathematical accuracy of its content, and the clarity of the task statements. In this process, feedback was obtained from two mathematics teachers and two academics with expertise in mathematics education. The experts were asked to evaluate the tasks in terms of their alignment with the generalized arithmetic framework, their ability to reflect the intended mathematical structures, their appropriateness for sixth- and seventh-grade students, the clarity of the instructions, and their comprehensibility for students. Based on the feedback received, the wording and instructions of some tasks were reviewed and revised as necessary, and the task set was finalized accordingly. The pairing rationale was established through task analysis: for each arithmetic and algebraic pair, the target relationship, representation, and response demand were identified explicitly. Thus, the matching used in this study is a theory-based structural matching intended for descriptive comparison rather than a claim of psychometric equivalence or strict task isomorphism. The task set consists of five arithmetic tasks (A1, A2, A3, A4, and A5) and five algebraic tasks (C1, C2, C3, C4, and C5) designed as structurally matched pairs. The matching was conceptual rather than strictly isomorphic: each pair was organized around a shared target mathematical relationship within generalized arithmetic, while the arithmetic and algebraic versions differed in representation and, in some pairs, also in the specific task demand. The tasks were structured taking into consideration three fundamental domains of generalized arithmetic. In this context, the properties of numbers were addressed through tasks A1 and A3 and their structurally matched counterparts C1

and C3; the properties of operations were addressed through tasks A2 and A5 and their structurally matched counterparts C2 and C5; and the relational meaning of the equal sign and equivalence were addressed through tasks A4 and C4. The preference for these domains was influenced by the fact that, within the scope of generalized arithmetic, the generalization of the properties of numbers and operations, as well as the structural treatment of equality and equivalence relations, holds a fundamental place in the development of algebraic thinking (Kaput, 2008; Kieran, 2022). Letters and symbolic representations were included in the algebraic counterparts of the tasks. Accordingly, the task pairs were used to compare students' performance and solution strategies across structurally related arithmetic and algebraic situations, without assuming that the two versions were identical in all respects. Table 1 summarizes the basis on which the task pairs were matched and makes explicit the main differences in task demand.

Table 1. *Structural Matching of the Arithmetic and Algebraic Task Pairs*

Task pair	Common mathematical structure	Arithmetic representation	Algebraic representation	Difference in task demand
A1–C1	Compensation and equivalence through balancing constant differences	Comparison of numerical expressions whose differences compensate for one another	Comparison of expressions containing a common variable and compensating constant differences	C1 requires interpreting the common variable and general equivalence, whereas A1 can be verified numerically.
A2–C2	Distributive/common-factor structure and equivalence of alternative forms	Evaluation of alternative solution approaches in a numerical contextual situation	Recognition of equivalence between algebraic forms such as $7m+9m$ and $16m$	A2 requires deciding between solution approaches, whereas C2 requires recognizing that two algebraic forms/strategies can both be valid.
A3–C3	Generalization of a number property	Reasoning from particular numerical products and their properties	Expressing the same type of number property in generalized form using a variable	C3 requires generalization beyond a specific numerical case.
A4–C4	Relational meaning of equality and equivalence	Solving a numerical open equality by attending to the relation between the two sides	Interpreting an equality/equivalence relation involving symbolic quantities	C4 places greater demand on interpreting the symbolic relation, whereas A4 can be solved through numerical operations.
A5–C5	Inverse/additive relationship among quantities	Reasoning about a concrete numerical relationship among distances/quantities	Representing the corresponding relationship symbolically with a variable	C5 requires constructing a general symbolic expression for an unknown/general quantity.

Thus, the term structurally matched is used in this study to indicate that each pair targets a common underlying relationship, not that the two tasks differ only in the use of arithmetic versus algebraic notation. This distinction is particularly important for A2–C2, where the shared distributive/equivalence structure is preserved but the response demand is not identical. The tasks were also structured as contextualized tasks. This preference is consistent with the skill- and process-oriented assessment and evaluation approach adopted in the Turkey Century Education Model (Türkiye Yüzyılı Maarif Modeli). In this model, importance is placed on students using their knowledge and skills in meaningful situations, and on evaluating learning outcomes not only in terms of the result reached but also in terms of the quality of the process (MEB, 2024). Accordingly, in the tasks, attention was paid not only to whether students reached the correct result but also to how they constructed their solutions and how they used mathematical relationships. In the qualitative phase of the study, task-based interviews were conducted in order to examine in greater detail the solution strategies students employed in the written tasks and the thinking processes underlying these strategies. Task-based interviews constitute an interview approach that enables a detailed examination of the thinking, reasoning, and problem-solving processes exhibited by the participant during interaction with the mathematical tasks presented by the researcher (Goldin, 2000; Maher & Sigley, 2020). In the interviews, students were asked to revisit the tasks, explain the solution methods they employed, and state the justifications for their responses. In this way, it was aimed to examine in detail, through students' own explanations, the thinking processes that could not be directly observed in the written responses. The interviews contributed, in particular, to determining how students used procedural/computational and relational/structural strategies and the uncertainties they experienced during the solution process. Detailed explanations regarding the data collection tools used in the study have been provided above, and the task set consisting of arithmetic and algebraic tasks is presented in the appendices.

The data collection process was carried out in two phases, in accordance with the explanatory sequential mixed-methods design of the study. In the first phase, the 10-item task set, consisting of five arithmetic tasks and five structurally matched algebraic tasks, was administered to 6th and 7th grade students in a classroom setting. Students were asked not only to state the results they had reached but also to present, in written form, the solution methods and thinking processes they employed. Following the administration, the student responses obtained were first classified in terms of performance status and the written solutions were then examined in terms of procedural/computational and relational/structural strategies. In the second phase, in order to explain in greater detail the different performance and solution strategy findings that emerged in the quantitative findings, task-based interviews were conducted with the six students identified. During the interview process, the same task set served as the common interview stimulus. Students were asked to revisit the tasks, explain the solution methods they employed, and state the justifications for their responses. Thus, the interviews followed a common task-based focus centered on students' explanations of how they reached their answers and which mathematical relationships they used. Their explanations were examined with particular attention to how they interpreted the mathematical relationships involved and to the basis of the strategy they used. In this way, aspects of the solution process that could not be directly observed in the written responses were examined through students' own explanations; the qualitative data were used primarily to elaborate the strategy patterns identified in the written data. The study was not designed to estimate task-order or priming effects. Accordingly, references made by students from one task to another were treated as qualitative evidence of perceived structural connections and were not interpreted causally.

2.4. Data Analysis

The data obtained in the study were analyzed in three phases, in accordance with the explanatory sequential mixed-methods design of the study. In the first phase, the performance status of students' responses to the arithmetic and algebraic tasks was determined; in the second phase, the written solutions were examined in terms of the strategies employed; and in the third phase, the distributions that emerged in the quantitative findings were elaborated upon through the interview data. Performance status and strategy type were treated as separate dimensions of analysis. Accordingly, a student's use of a relational/structural approach was not necessarily interpreted as an indication that the student had reached the correct result.

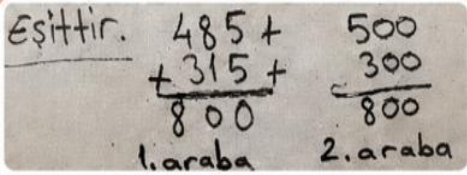
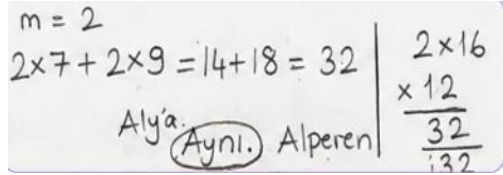
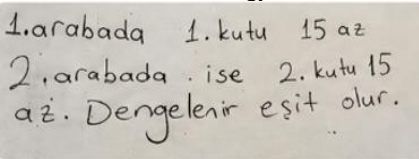
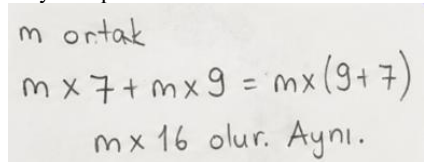
In the first phase, the written responses given to the 10-item data collection instrument, consisting of five arithmetic and five structurally matched algebraic tasks, were examined. Student responses were classified into three categories for each task: correct, incorrect, and blank. In addition, whether or not students provided explanations for their responses was also examined separately. The data obtained were analyzed using frequency and percentage values. No inferential statistical tests were conducted; therefore, differences between grade levels and task types are interpreted as descriptive patterns rather than as statistically established effects or developmental change. The analyses were conducted taking into account the 6th and 7th grade levels, as well as the domains of the properties of numbers, the properties of operations, and the equal sign/equivalence. The rates of correct responses in the arithmetic and algebraic tasks that were paired with one another were also compared, and the findings that emerged according to task type and grade level were presented through tables and graphs. In this way, it was determined how students' performance status varied according to grade level, task type, and the mathematical structure addressed.

In the second phase, students' written solutions were analyzed in terms of the mathematical approaches they employed in reaching a result. Strategies were classified into two main categories: procedural/computational and relational/structural. In the coding process, the focus was not solely on the result reached, but on which mathematical relationship or operation the student had based their solution on. Procedural/computational strategies refer to solution approaches that focus on the operational steps to be carried out rather than on the underlying structure and relationships of the mathematical expression. In such solutions, carrying out operations separately, performing algorithmic computation, assigning specific values, and comparing the results obtained come to the fore, whereas the use of mathematical relationships and structural properties remains limited (Radford, 2022). For this reason, a solution's reaching the correct result was not, on its own, taken as an indication that the student had recognized the mathematical structure (Linchevski & Livneh, 1999). Relational/structural strategies encompass approaches in which the student recognizes the properties, relationships, and equivalences among numbers, operations, and mathematical expressions, and actively uses these structures during the solution process. In such solutions, identifying the common structure, making use of the properties of operations, decomposing and recombining expressions, and constructing equivalent expressions come to the fore (Carpenter et al., 2005; Subramaniam & Banerjee, 2011). Going beyond the computational process to recognize the mathematical structure and to justify the solution on the basis of these structures is considered one of the fundamental indicators of relational/structural thinking (Kieran, 2022). Strategy coding was carried out independently of performance status. Therefore, both correct and incorrect responses could be coded as either procedural/computational or relational/structural, depending on the nature of the approach used during the solution process. No strategy coding was performed for blank responses. This distinction, in

particular, made it possible to render visible the thinking processes of students who recognized a relational/structural relationship but were unable to reach a correct and complete solution. In the interview analysis, solutions marked as partial or hesitant were included within the relational/structural category only when the student explicitly attended to a relevant mathematical relationship or structural property but did not articulate or carry that relationship through in a fully developed way. The asterisk used in the interview strategy table therefore indicates a qualitative degree of completeness within the relational/structural category; it does not constitute a third strategy category. The data obtained from the strategy analysis were presented with frequency and percentage values according to task, grade level, and task type. Strategy percentages were calculated over all students at the relevant grade level, with blank responses retained in the denominator and reported as a separate category. In addition, graphs were used in order to make the distribution of procedural/computational, relational/structural, and blank responses across the tasks more visible. In cases where more than one approach was observed together within a single solution, the approach that the student used as the primary justification in reaching the result was taken as the basis. In cases where a change in strategy was observed during the solution process in the interviews, this change was additionally described qualitatively.

In addition to the structure, scope, and functions of the data collection tools used in the study, information regarding the sample solutions and coding approaches used in the analysis of the data obtained from these tools is presented in Table 2.

Table 2. Coding Criteria and Examples Used in the Analysis of Solution Strategies

	A1 – Arithmetic task example	C2 – Algebraic task example
Procedural/ Computational	<p>The student calculated the two expressions separately, reaching the results $485 + 315 = 800$ and $500 + 300 = 800$, and made a decision based on the equality of the results. For this reason, the solution was evaluated as a procedural/computational strategy, based on direct calculation rather than on the use of a relational structure.</p> 	<p>The student assigns a specific value in place of m. For example, for $m = 2$, the student performs the operations $2 \times 7 + 2 \times 9 = 32$ and $2 \times 16 = 32$ and states that the expressions are equal. This verification, carried out through a specific value, does not explain why the expressions are equivalent for every value of m.</p> 
Relational/ Structural	<p>Rather than calculating the operations separately, the student recognized the equivalent structure present in the two cases and explained the equality by stating that the expressions on both sides represented the same value. For this reason, the solution was evaluated as a relational/structural strategy.</p> 	<p>The student recognizes the common factor m in the expression $7m + 9m$ and establishes the relationship $7m + 9m = m(7+9) = 16m$. The student explains structurally that the expressions are equivalent not only for specific values but for all values of m.</p> 

In the third phase, the task-based interviews conducted with the six students were analyzed in order to explain, in greater detail, the performance and strategy findings that emerged in the quantitative analyses. In the interviews, how students approached the tasks, how they explained the solution methods they employed, and how they justified mathematical relationships were examined. In the analysis of the interview data, the procedural/computational and relational/structural strategy framework used in the written solutions was preserved. At the same time, the interviews also made it possible to identify thinking processes that were not clearly observable in the written responses. In particular, attention was paid to students' changing strategies during the solution process, their partial recognition of a structural relationship, their hesitations, how they made sense of the variable, and how they dealt with equivalence relationships. The interview findings were presented not only through general descriptions but also through direct excerpts of students' dialogue. The selected excerpts were tabulated so as to exemplify students' procedural/computational and relational/structural modes of thinking, and each excerpt was interpreted in terms of the type of strategy involved. In this way, the general findings presented through frequencies, percentages, and graphs in the quantitative analyses were elaborated upon with the qualitative data obtained from students' individual solution processes.

2.5. Procedures to Support Validity and Reliability

In order to support the consistency and transparency of the analysis, care was taken to ensure that the data collection and analysis processes were carried out in an explicit, systematic, and traceable manner. The procedural/computational and relational/structural categories used in the analysis of students' solution strategies were defined on the basis of the relevant theoretical framework, and explicit analysis criteria were established for each category (Carpenter et al., 2005; Kieran, 2022; Linchevski & Livneh, 1999; Radford, 2022; Subramaniam & Banerjee, 2011). Grounding the strategy categories in theoretical foundations supported the conceptual consistency and validity of the analytical framework. In addition, in order to make explicit the criteria on which coding decisions were based, sample student solutions for the arithmetic and algebraic tasks were presented, along with an account of how these solutions were evaluated in terms of strategy. In this way, an effort was made to enhance the transparency and confirmability of the analysis process.

In the analysis process, performance status and solution strategy were treated as two distinct dimensions. Student responses were first classified as correct, incorrect, or blank. Correct and incorrect responses were then coded as procedural/computational or relational/structural according to the nature of the solution approach employed. This distinction made it possible to differentiate between obtaining a correct result and recognizing and using the underlying mathematical structure in the solution. It also allowed responses in which students demonstrated awareness of a structural relationship but did not reach a correct or complete solution to be represented in the analysis. The coding process was conducted by the researcher using predefined criteria for procedural/computational and relational/structural strategies. Responses that were difficult to classify, reflected characteristics of more than one strategy, or raised uncertainty during coding were re-examined together with a second researcher/field expert, and coding decisions were finalized through consensus. In addition, selected student responses were reviewed by the second researcher/field expert to examine the applicability and consistency of the coding criteria. Because the entire dataset was not independently coded by two separate coders, no quantitative inter-rater reliability coefficient was calculated. Instead, the explicit definition of the coding categories, the consistent application of the same criteria across responses, and the review of ambiguous and selected cases by a second researcher/field expert were used to enhance the transparency and consistency of the analysis.

In reporting the quantitative findings, frequency and percentage values were used, and the findings were supported with tables and graphs. The arithmetic and algebraic tasks that were paired with one another were compared according to grade level and task domain, thereby ensuring that the performance and strategy findings were presented in a systematic and comparable manner. Presenting the findings through different modes of representation supported the traceability of the analysis process and the auditability of the findings. Nevertheless, considering the sampling characteristics of the study, the contextual and sampling-related limitations of the research need to be taken into account when generalizing the findings obtained to broader student populations.

In the presentation of the qualitative findings, direct dialogue excerpts from students' task-based interviews were included in order to make explicit the basis for the researcher's interpretations. Presenting students' statements together with the researcher's interpretations ensured that the inferences drawn were directly linked to the data, thereby supporting the credibility and confirmability of the qualitative findings (Lincoln & Guba, 1985). In addition, the fact that the task-based interviews were conducted using the same task set employed in the quantitative phase made it possible to examine, in greater detail, the performance and strategy differences that emerged in the quantitative findings through students' solution processes and explanations.

In accordance with the explanatory sequential mixed-methods structure of the study, the quantitative findings obtained from the written tasks and the qualitative findings obtained from the task-based interviews were related to one another during the analysis and interpretation phase. The performance and strategy findings identified in the quantitative phase were elaborated upon through students' solution processes and explanations in the qualitative phase. In this way, attention was focused not only on group-level distributions but also on how these distributions manifested themselves in students' individual thinking processes. The integration of the quantitative and qualitative findings in line with the research questions supported the validity and holistic interpretation of the mixed-methods inferences (Creswell & Plano Clark, 2018). Through this integration, the ways in which the two data sources supported, explained, or diverged from one another were evaluated together.

3. Findings

3.1. Distribution of 6th and 7th Grade Students' Performance Status in Arithmetic and Algebraic Tasks According to Task Type and Grade Level

The findings obtained in order to reveal how students' performance status in the arithmetic and algebraic tasks was distributed according to task type and grade level are presented in Table 3.

Table 3. Performance Status of 6th and 7th Grade Students in Arithmetic and Algebraic Tasks

Types of Task	Grade	Arithmetic Task					Algebraic Task		
		C	InC	B		C	InC	B	
Properties of Operations [PoO]	6	A2	46 (%51,1)	25 (%27,8)	19 (%21,1)	C2	34 (%37,8)	31 (%34,4)	25 (%27,8)
	7	A2	41 (%68,3)	10 (%16,7)	9 (%15,0)	C2	31 (%51,7)	12 (%20,0)	17 (%28,3)
	6	A5	12 (%13,3)	66 (%73,3)	12 (%13,3)	C5	6 (%6,7)	58 (%64,4)	26 (%28,9)
	7	A5	33 (%55,0)	23 (%38,3)	4 (%6,7)	C5	12 (%20,0)	36 (%60,0)	12 (%20,0)
Properties of Numbers [PoN]	6	A3	47 (%52,2)	31 (%34,4)	12 (%13,3)	C3	17 (%18,9)	48 (%53,3)	25 (%27,8)
	7	A3	46 (%76,7)	11 (%18,3)	3 (%5,0)	C3	19 (%31,7)	23 (%38,3)	18 (%30,0)
	6	A1	71 (%78,9)	6 (%6,7)	13 (%14,4)	C1	46 (%51,1)	33 (%36,7)	11 (%12,2)
	7	A1	57 (%95,0)	2 (%3,3)	1 (%1,7)	C1	42 (%70,0)	9 (%15,0)	9 (%15,0)
Properties Related to Equality [PRE]	6	A4	67 (%74,4)	13 (%14,4)	10 (%11,1)	C4	49 (%54,4)	23 (%25,6)	18 (%20,0)
	7	A4	50 (%83,3)	5 (%8,3)	5 (%8,3)	C4	43 (%71,7)	9 (%15,0)	8 (%13,3)

Note. C: Correct, InC: Incorrect, B: Blank and PoO: Properties of Operations, PoN: Properties of Numbers, PRE: Properties Related to Equality

When Table 3 is examined, it can be seen that 7th grade students had higher correct response rates than 6th grade students on all tasks. Nevertheless, at both grade levels, the correct response rates for the arithmetic tasks were higher than those for the corresponding algebraic tasks. When the five tasks are considered together, the average correct response rate for the arithmetic tasks among 6th grade students was 54.0%, whereas it was 33.8% for the algebraic tasks. In the 7th grade, these rates were 75.7% and 49.0%, respectively. Thus, correct-response rates were higher in the 7th-grade group than in the 6th-grade group for both task types; however, the difference between the arithmetic and algebraic tasks persisted in both groups. Differing patterns of performance emerged in the tasks related to the properties of numbers. A1 is the task with the highest correct response rate at both grade levels. In this task, the correct response rate was 78.9% in the 6th grade and 95.0% in the 7th grade. In C1, the structurally matched algebraic counterpart of A1, these rates dropped to 51.1% and 70.0%, respectively. In the A3–C3 pair, the difference between the arithmetic and algebraic tasks is more pronounced. While the correct response rate for A3 was 52.2% in the 6th grade and 76.7% in the 7th grade, these rates remained at 18.9% and 31.7% for C3. A difference in favor of the arithmetic task is also observed in the A2–C2 task pair, related to the properties of operations. In the 6th grade, the correct response rate for A2 was 51.1%, whereas this rate was 37.8% for C2. In the 7th grade, these rates were 68.3% and 51.7%, respectively. The A5–C5 pair, which falls within the scope of the properties of operations, comprises the tasks in which the lowest performance rates were generally observed. In the 6th grade, the correct response rate for A5 was 13.3%, and only 6.7% for C5. In the 7th grade, the correct response rate for A5 rose to 55.0%, but remained at 20.0% for C5. Relatively higher performance rates were observed in the tasks related to the properties of equality. In task A4, the correct response rate was 74.4% in the 6th grade and 83.3% in the 7th grade. In task C4, these rates were determined to be 54.4% and 71.7%, respectively. It is particularly notable that, in the 7th grade, the difference between A4 and C4 is more limited compared to some of the other task pairs. In order to examine in more detail how the overall performance distributions that emerged in the table varied on the basis of the tasks with their arithmetic and algebraic counterparts, the correct response rates were compared according to tasks in Figure 1.

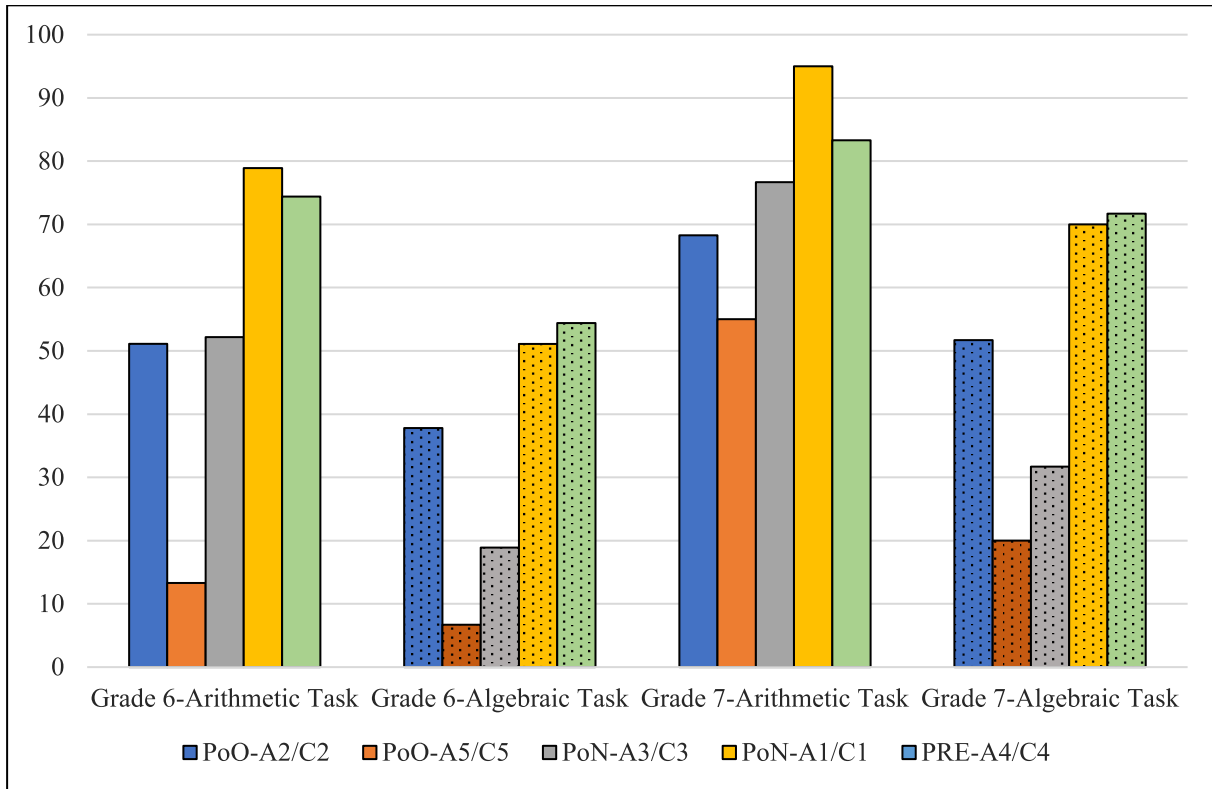


Figure 1. Correct Response Rates of 6th and 7th Grade Students in Arithmetic and Algebraic Tasks

When Figure 1 is examined, it can be seen that, at both grade levels, the correct response rates for the arithmetic tasks were higher than those for the corresponding algebraic tasks across all task pairs. Nevertheless, the magnitude of the difference between the arithmetic and algebraic tasks varies according to the mathematical structure involved in the task. In the 6th grade, there is a 13.3 percentage point difference between tasks A2–C2. This difference rises to 27.8 percentage points for the A1–C1 pair and to 33.3 percentage points for the A3–C3 pair. While there is a 20.0 percentage point difference between A4–C4, the difference between A5–C5 is 6.6 percentage points. The relatively small difference between A5 and C5 stems from the fact that the correct response rates for both tasks are quite low. The same general pattern continues in the 7th grade. The difference between tasks A2–C2 is 16.6 percentage points, and the difference between A1–C1 is 25.0 percentage points. The most pronounced difference emerged in the A3–C3 task pair, where the gap reached 45.0 percentage points. The difference in the A5–C5 task pair is 35.0 percentage points. The fact that the difference between A4–C4 is 11.6 percentage points indicates that, in the 7th grade, the difference between the arithmetic and algebraic forms in the tasks related to equality is more limited compared to many of the other task pairs.

When Table 3 and Figure 1 are considered together, correct-response rates were generally higher in the 7th-grade group than in the 6th-grade group; however, within both grade groups, performance was lower on the algebraic task in every structurally matched pair. This decrease is particularly pronounced in the A3–C3 and A5–C5 task pairs. In contrast, the high performance rates observed in tasks A1 and A4 continue at a relatively high level in their algebraic counterparts, C1 and C4, compared to many of the other algebraic tasks. The interview data were not used to construct a second performance classification. Instead, they were used as explanatory qualitative evidence for examining how students described, justified, and sometimes revised the strategies evident in their solutions. This preserves the correct–incorrect–blank classification as the performance measure used in the quantitative phase and uses the interviews primarily to illuminate strategy and reasoning.

3.2. Distribution of the Solution Strategies Used by 6th- and 7th-Grade Students in Arithmetic and Algebraic Tasks According to Task Type and Grade Level

The distribution of the procedural/computational and relational/structural solution strategies used by 6th-grade and 7th-grade students in arithmetic and algebraic tasks according to task type and grade level is presented in Table 4.

Table 4. *Solution Strategies Used by 6th- and 7th-Grade Students in Arithmetic and Algebraic Tasks*

		Arithmetic Task					Algebraic Task					
Grade/ Types of Task		Procedural/ Computational			Relational/ Structural			Procedural/ Computational			Relational/ Structural	
		C	InC	B	C	InC		C	InC	B	C	InC
6/PoO	A2	45 (%50,0)	25 (%27,8)	19 (%21,1)	1 (%1,1)	0 (%0,0)	C2	32 (%35,6)	29 (%32,2)	25 (%27,8)	2 (%2,2)	2 (%2,2)
	A5	12 (%13,3)	66 (%73,3)	12 (%13,3)	0 (%0,0)	0 (%0,0)	C5	6 (%6,7)	56 (%62,2)	26 (%28,9)	0 (%0,0)	2 (%2,2)
6/PoN	A3	46 (%51,1)	31 (%34,4)	12 (%13,3)	1 (%1,1)	0 (%0,0)	C3	16 (%17,8)	46 (%51,1)	25 (%27,8)	1 (%1,1)	2 (%2,2)
	A1	70 (%77,8)	6 (%6,7)	13 (%14,4)	1 (%1,1)	0 (%0,0)	C1	44 (%48,9)	29 (%32,2)	11 (%12,2)	2 (%2,2)	4 (%4,4)
6/PRE	A4	66 (%73,3)	13 (%14,4)	10 (%11,1)	1 (%1,1)	0 (%0,0)	C4	48 (%53,3)	21 (%23,3)	18 (%20,0)	1 (%1,1)	2 (%2,2)
7/PoO	A2	38 (%63,3)	10 (%16,7)	9 (%15,0)	3 (%5,0)	0 (%0,0)	C2	17 (%28,3)	9 (%15,0)	17 (%28,3)	14 (%23,3)	3 (%5,0)
	A5	31 (%51,7)	23 (%38,3)	4 (%6,7)	2 (%3,3)	0 (%0,0)	C5	8 (%13,3)	25 (%41,7)	12 (%20,0)	4 (%6,7)	11 (%18,3)
7/PoN	A3	42 (%70,0)	11 (%18,3)	3 (%5,0)	4 (%6,7)	0 (%0,0)	C3	12 (%20,0)	19 (%31,7)	18 (%30,0)	7 (%11,7)	4 (%6,7)
	A1	54 (%90,0)	2 (%3,3)	1 (%1,7)	3 (%5,0)	0 (%0,0)	C1	22 (%36,7)	8 (%13,3)	9 (%15,0)	20 (%33,3)	1 (%1,7)
7/PRE	A4	47 (%78,3)	5 (%8,3)	5 (%8,3)	3 (%5,0)	0 (%0,0)	C4	28 (%46,7)	8 (%13,3)	8 (%13,3)	15 (%25,0)	1 (%1,7)

When Table 4 is examined, it can be seen that procedural/computational strategies were common at both grade levels, whereas the use of relational/structural strategies varied according to task type and grade level. This difference becomes particularly evident in the algebraic tasks. Among 6th grade students, the total rate of use of relational/structural strategies was 0.9% in the arithmetic tasks and 4.0% in the algebraic tasks. In the 7th grade, these rates were 5.0% and 26.7%, respectively. Thus, the difference between the grade groups in the observed use of relational/structural strategies was particularly pronounced in the algebraic tasks. The 6th grade students' solutions in the arithmetic tasks were largely procedural/computational in nature. In tasks A1, A2, A3, and A4, correct relational/structural solutions were observed at a rate of only 1.1%, whereas no relational/structural strategy was observed in task A5. In these tasks, students mostly carried out numerical operations directly and attempted to reach the result through calculation. In the 6th grade's algebraic tasks, the use of relational/structural strategies was somewhat higher than in the arithmetic tasks, but remained limited. The total rate of use of relational/structural strategies reached its highest value among the algebraic tasks at this grade level in C1, at 6.7%. This rate was 4.4% in C2, 3.3% in C3 and C4, and 2.2% in C5. However, it is observed that relational/structural strategies did not always result in a correct response. For example, in C1, 2.2% of students using a relational/structural strategy gave a correct response, while 4.4% gave an incorrect response. Procedural/computational strategies also retained their dominance in the 7th grade students' arithmetic tasks. The rate of use of relational/structural strategies was 5.0% in A2, A1, and A4; 6.7% in A3; and 3.3% in A5. Accordingly, although the two grade groups differed in their observed rates, the primary solution approach in the arithmetic tasks was largely procedural/computational in both groups. In contrast, relational/structural strategies were observed at higher rates in the 7th-grade group's algebraic tasks. The total rate of use of these strategies was 35.0% in C1, 28.3% in C2, 26.7% in C4, 25.0% in C5, and 18.3% in C3. In particular, in tasks C1, C2, and C4, students appear to have tended toward using the commonality of variables, equivalence, and equality relationships. It is also notable that the higher observed use of relational/structural strategies in the 7th-grade algebraic tasks was not always accompanied by a correct response. This situation is observed particularly in the 7th grade algebraic tasks. In C5, 6.7% of students gave a correct response using a relational/structural strategy, while 18.3% gave an incorrect response despite using the same type of strategy. Similarly, in C2, 23.3% of students using a relational/structural strategy gave a correct response and 5.0% gave an incorrect response; in C3, the corresponding rates were 11.7% correct and 6.7% incorrect. In contrast, no instance was observed in the arithmetic tasks in which a relational/structural strategy resulted in an incorrect response. Overall, Table 4 shows that the procedural/computational approach was the primary solution method in both grade groups, whereas relational/structural strategies were observed at markedly higher rates in the 7th-grade group, particularly in the

algebraic tasks. It is also observed that there is no one-to-one correspondence between the use of a relational/structural approach and reaching the correct result.

In order to make the general task-based findings regarding strategy use, presented in detail together with correct and incorrect responses in Table 4, more visible, the total rates of use of the procedural/computational and relational/structural strategies are compared in Figure 2. Correct and incorrect responses were combined on the basis of strategy type, while blank responses are shown as a separate category.

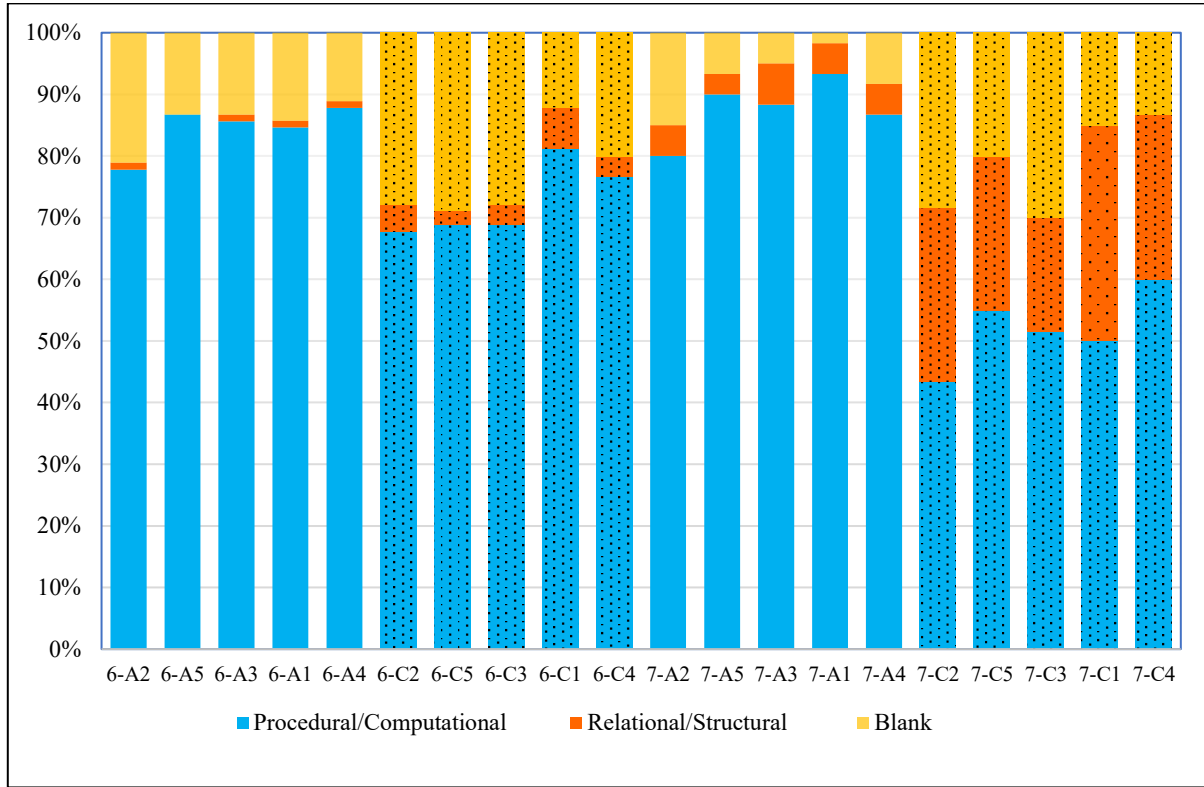


Figure 2. Task-Based Distribution of the Solution Strategies Used by 6th and 7th Grade Students in Arithmetic and Algebraic Tasks

When Figure 2 is examined, it can be seen that procedural/computational strategies were clearly dominant in all of the 6th grade arithmetic tasks. The rate of use of this strategy was 77.8% in A2, 86.7% in A5, 85.6% in A3, 84.4% in A1, and 87.8% in A4. The use of relational/structural strategies, on the other hand, remained at only 1.1% except in A5. In the 6th grade algebraic tasks, procedural/computational strategies remained dominant as well; however, relational/structural strategies were found to be more visible than in the arithmetic tasks. It is particularly notable that the use of relational/structural strategies in C1 reached 6.7%. Nevertheless, these rates remained lower in C2, C3, C4, and C5. It can also be seen that blank responses in the algebraic tasks were generally higher than in the arithmetic tasks. In the 7th grade arithmetic tasks, the use of procedural/computational strategies ranged between 80.0% and 93.3%. The rate of relational/structural strategies, in contrast, remained between 3.3% and 6.7%. In the algebraic tasks, however, the distribution changes markedly. The total rate of relational/structural strategies reaches 35.0% in C1, 28.3% in C2, 26.7% in C4, and 25.0% in C5. Thus, it can be seen that, at the 7th grade level, the algebraic tasks elicited relational/structural solution strategies to a greater extent than the arithmetic tasks.

The figure also draws attention to the high rates of blank responses in some of the algebraic tasks. This rate was 28.9% for C5 and 27.8% for C2 and C3 in the 6th grade, and 30.0% for C3 and 28.3% for C2 in the 7th grade. Accordingly, the algebraic tasks differ from the arithmetic tasks not only in terms of the type of strategy used but also in terms of students' capacity to produce a solution. When Table 4 and Figure 2 are considered together overall, relational/structural strategies were observed more frequently in the 7th-grade group than in the 6th-grade group, with the difference concentrated mainly in the algebraic tasks. In the arithmetic tasks, the procedural/computational approach remains strongly preserved at both grade levels. In the algebraic tasks, on the other hand, it is observed that 7th grade students in particular tend to make greater use of mathematical structures and relationships.

In order to examine in more detail how these general strategy distributions identified in the quantitative findings were reflected in students' individual solution processes, the strategies used by the six students who participated in the interviews in the arithmetic and algebraic tasks were evaluated on a task-by-task basis. The

interview data were treated not as a direct counterpart of the quantitative frequencies, but as qualitative findings explaining the nature of students' solution approaches. The strategy patterns obtained are presented in Table 5.

Table 5. *General Strategy Distributions of Students Participating in the Interviews*

Grade/ Students	Arithmetic Task		Algebraic Task	
	Procedural/ Computational	Relational/ Structural	Procedural/ Computational	Relational/ Structural
Grade 6	Deniz	A1, A3, A4, A5	A2	C4, C5
	Miray	A1, A3, A4, A5	A2	C1, C3, C4, C5
	Aleyna	A1, A3, A4	A2*, A5	C1*, C2*, C3, C4, C5
Grade 7	Zeynep	–	A1, A2, A3, A4, A5	–
	Bahar	A1, A3, A4, A5	A2	C5
	Ali	A1, A3, A4	A2, A5*	–

Note. The * symbol indicates solutions in which relational/structural thinking emerged in a partial, hesitant, or not-yet-fully-structured manner.

When Table 5 is examined, it can be seen that procedural/computational strategies were more prominent in the 6th grade students' arithmetic tasks. Deniz and Miray used a procedural/computational strategy in four of the five arithmetic tasks. In Aleyna's case, relational/structural thinking emerged in A2 and A5, alongside the procedural approach. However, this thinking exhibited a hesitant pattern in A2. In the 6th grade's algebraic tasks, on the other hand, more varied strategy profiles are observed among the students. Deniz used a relational/structural strategy in C1, C2, and C3, while turning to a procedural approach in C4 and C5. In Miray's case, the procedural approach continued in C1, C3, C4, and C5, while relational/structural thinking emerged partially in C2. In Aleyna's case, all of the algebraic tasks were coded as relational/structural, although this thinking remained at a partial or hesitant level in C1 and C2. These findings indicate that relational/structural thinking can emerge among 6th grade students, but is not consistently generalized across students and tasks. In the interviews with the 7th grade students, relational/structural strategies exhibited a more pronounced pattern. Zeynep used a relational/structural strategy in all of the arithmetic and algebraic tasks. While a procedural approach was dominant in Bahar's arithmetic tasks, relational/structural strategies were observed in four of the algebraic tasks. In Ali's arithmetic solutions, both types of strategy were observed together, whereas all of the algebraic tasks were coded as relational/structural. Nevertheless, relational thinking remained at a partial level in C3 and C5. Overall, the interview findings are consistent with the quantitative pattern in which relational/structural strategies were observed more frequently among the seventh-grade participants, particularly in the algebraic tasks. This consistency should be understood as qualitative evidence that helps explain selected aspects of the observed pattern, rather than as statistical validation of the quantitative findings or evidence that the interview findings apply to the full sample.

The general strategy pattern obtained from the interviews indicates that students can exhibit different solution profiles even within the same grade level. In particular, it was observed that, among 6th grade students, procedural/computational and relational/structural approaches could vary from task to task, and that relational thinking emerged in a partial or hesitant manner in some cases. Among the 7th-grade interviewees, relational/structural strategies were observed more prominently and consistently. In order to render this differentiation visible through students' own explanations, selected interview excerpts from Miray (Table 6), Aleyna, and Zeynep, who exhibited different strategy profiles, were examined.

Table 6. *Miray's Interview Excerpts and Strategy Analysis on Selected Tasks*

Task	Interview Excerpt	Strategy	Evaluation
A1	"I needed to find the sum of both... Then I saw that they were equal."	Procedural/ Computational	Calculated the two sums separately and compared the results. Determined the equality on the basis of the numerical result.
A2	"There were 6 and 5 at each table. If we add these and multiply by 72, we can find it."	Relational/ Structural	Recognized the structure $72(6+5)$ and developed the solution by using the common structure.
A4	"I add 81 and 24, and then subtract 78 from this sum."	Procedural/ Computational	Reached the unknown through addition and subtraction operations, rather than directly using the relationship between the two sides of the equality.
C1	"I subtract 85 from 315 and find the variable n."	Procedural/ Computational	Treated n not as a variable quantity common to the two expressions, but as an unknown to be found.

Task	Interview Excerpt	Strategy	Evaluation
C2	"If we add 7 and 9, that's 16... I thought of it as m times 16... The two seemed the same to me."	Relational/ Structural (Limited)	Moved toward recognizing the relationship between $7m+9m$ and $16m$, but was unable to transform this explanation into a definite and stable structural justification.
C5	"At first I added them all up... but I did it wrong."	Procedural/ Computational (Limited)	Attempted to combine the given quantities through operations, but was unable to establish the general relationship between the quantities.

Miray's solution process indicates that the procedural/computational approach remained dominant. Nevertheless, in A2 she was able to directly use the common structure, and in C2 she approached recognizing the equivalence. However, the hesitation observed in C2 and the procedural approach seen in C1 and C5 indicate that relational/structural thinking had not yet been sustained consistently across tasks for Miray. In contrast to this pattern observed in Miray - predominantly procedural but occasionally moving toward relational/structural thinking - Aleyna's solution processes show a more pronounced combined use of both types of strategy. Aleyna's interview excerpts on selected tasks and the strategy analyses of these excerpts are presented in Table 7.

Table 7. *Aleyna's Interview Excerpts and Strategy Analysis on Selected Tasks*

Task	Interview Excerpt	Strategy	Evaluation
A1	"I added both cars and found the result to be equal."	Procedural/ Computational	Calculated the two situations separately and compared them, reaching the equality as a result of the numerical operation.
A3	" 18×45 , we found 810... we did 45×12 ... then adding 810 and 540 gives 1350."	Procedural/ Computational	First carried out the multiplication and addition operations, and turned to the properties of numbers only after obtaining the result of the operation.
A4	"We will find 81 and 24... and we subtract 78 from 115. That leaves 37."	Procedural/ Computational	Attempted to solve the equality not through a balance relationship, but through successive addition and subtraction operations.
A5	"If it goes 61 km from C to B, then to get back it needs to go another 61 km."	Relational/ Structural	Recognized the inverse relationship between the distances and based the solution on this relationship.
C1	" $85+315$ makes 400... since I don't know the n's, it can't always be equal."	Relational/ Structural (Limited)	Recognized that the constant parts were equal, but was unable to generalize that the common variable preserved the equality.
C3	"Multiples of 50 are always even, teacher..."	Relational/ Structural	Generalized the property of numbers related to multiples of 50, rather than calculating specific values one by one.

Aleyna's interview excerpts clearly show that procedural and relational/structural strategies can be observed together within the same student. While the procedural approach came to the fore in A1, A3, and A4, mathematical relationships were used directly in A5 and C3. In C1, the structural relationship was partially recognized, but generalization was not achieved. For this reason, Aleyna's profile shows a task-dependent shift between procedural and relational thinking. Whereas Aleyna is seen to use procedural and relational/structural strategies together in a task-dependent manner, it is notable that, in Zeynep's solutions, relational/structural thinking is sustained in a more stable and consistent manner. In order to demonstrate this differentiation more clearly, Zeynep's interview excerpts on selected tasks and the strategy analyses of these excerpts are presented in Table 8.

Table 8. *Zeynep's Interview Excerpts and Strategy Analysis on Selected Tasks*

Task	Interview Excerpt	Strategy	Evaluation
A1	"One car is short by 15, the other is over by 15; these cancel each other out and become equal."	Relational/ Structural	Used the fact that the numerical differences balanced each other out, without calculating the sums separately.
A2	"32 has no meaning here; Ece found the total at one table and multiplied it by 72."	Relational/ Structural	Identified the common structure that was meaningful in the task and explained the solution on the basis of this structure.
A3	"I didn't fully multiply 45×18 ; I looked at the units digit, the result ends in 0."	Relational/ Structural	Drew an inference about the nature of the result by making use of number properties, without completing the operation.
C1	"It's the same as question A1; on one side it's 15 more, on the other side it's 15 less."	Relational/ Structural	Focused on the fact that the constant differences balanced each other out in the expressions containing variables.

Task	Interview Excerpt	Strategy	Evaluation
C2	"Both Alya and Alperen are correct; one found it separately, the other used the total at one table."	Relational/ Structural	Exhibited the idea of equivalence by recognizing that different algebraic representations represented the same quantity.
C5	"If the total distance is n , then subtracting the distances already covered, the remaining part is $n-147$."	Relational/ Structural	Expressed the relationship between the quantities symbolically and used the variable as a general quantity.

The absence of a procedural example in Zeynep's table is not a deficiency, but precisely what was expected. In the main strategy table as well, all of Zeynep's arithmetic and algebraic tasks were coded as relational/structural. The six selected excerpts show how this strategy was sustained across different tasks involving number properties, common structure, balancing, equivalence, and relationships involving variables.

When the selected interview excerpts of the three students are considered together, it can be seen that students' solution strategies vary depending on both the student and the structure of the task. In Miray's case, the procedural/computational approach remained dominant, with relational/structural thinking emerging only in certain tasks and, at times, in a limited manner. In Aleyna's case, it was determined that procedural and relational/structural strategies were used together in a task-dependent manner, with procedural solutions coming to the fore in some tasks while mathematical relationships were used directly in others. In contrast, in Zeynep's selected excerpts, relational/structural thinking was observed to be sustained in a more stable and consistent manner. This pattern indicates that, in the interview data, different strategy profiles emerged among students—predominantly procedural, transitional, and more consistently relational/structural. In particular, balancing, common structure, equivalence, the generalization of number properties, and the use of relationships between variables stood out as clear indicators of relational/structural thinking. Nevertheless, recognizing a structural relationship did not always mean reaching a correct and complete solution. This situation was observed clearly, in particular, in some of Miray's and Aleyna's tasks. Accordingly, the interview findings reveal that the use of strategies can vary not only according to grade level, but also according to the student's solution approach and the mathematical structure of the task.

4. Discussion

The findings related to the first research question showed that seventh-grade students had higher correct-response rates than sixth-grade students across all tasks; however, at both grade levels, arithmetic tasks were answered correctly at higher rates than their corresponding algebraic tasks. When the five tasks were considered together, correct-response rates were 54.0% for arithmetic and 33.8% for algebraic tasks in Grade 6, and 75.7% and 49.0%, respectively, in Grade 7. Importantly, the compared tasks were designed as structurally matched arithmetic and algebraic pairs rather than as strictly identical tasks. The pairs shared a target mathematical relationship, but in some cases also differed in task demand, such as deciding between alternative solutions versus judging the equivalence of two valid expressions. Therefore, the observed differences should be interpreted as descriptive differences across structurally matched arithmetic and algebraic tasks rather than as effects of representation alone.

This finding is consistent with Pitta-Pantazi, Chimoni, & Christou (2025), who similarly reported higher performance on arithmetic than on corresponding algebraic tasks among sixth-grade students. They argued that students may experience difficulty when moving from expressions involving known quantities to letter-symbolic expressions containing both known and unknown quantities, particularly when they rely heavily on computation. Similar difficulties have also been documented by Liebenberg, Linchevski, Sasman, & Olivier (1999) and Linchevski and Livneh (1999). The present study extends this descriptive pattern by showing that it was evident among both Grade 6 and Grade 7 students in the present sample.

At the same time, the magnitude of the arithmetic–algebra difference varied across mathematical structures. The largest differences were observed in A3–C3 and, particularly in Grade 7, A5–C5, whereas the A4–C4 difference was comparatively smaller. This suggests that the distinction between arithmetic and algebra cannot be attributed solely to the presence of letters. Rather, the meaning of the represented quantities and the demands for generalization, equivalence, and structural transformation also appear to be important. In A3–C3, for example, students were more successful when reasoning about specific numbers than when the same numerical property had to be expressed and justified in a generalized algebraic form. This finding is consistent with Kaput's (2008) view of generalized arithmetic and Kieran's (2022) structural-thinking dimension, both of which emphasize generalizing properties of numbers, operations, and equivalence beyond particular numerical cases.

The A5–C5 pattern also illustrates the additional demands associated with algebraic representation. In Grade 7, the correct-response rate decreased from 55.0% in A5 to 20.0% in C5. The interview data helped to elaborate this descriptive pattern by showing that some students were able to notice relevant relationships in the algebraic task but were not always able to carry these relationships through to a complete and correct solution. In other

words, the lower performance observed in C5 did not necessarily mean that students failed to recognize the underlying relationship; in some cases, their explanations suggested partial structural awareness that was not successfully translated into a correct algebraic response. This finding is consistent with perspectives emphasizing that algebraic thinking involves not only attention to structure but also analytical reasoning with unknown quantities. Filloy and Rojano (1989) described a “cut” between arithmetic and algebra when arithmetic strategies are no longer sufficient, while Herscovics and Linchevski (1994) referred to a related “cognitive gap.” Kieran (2022) similarly identifies analyticity as a central dimension of algebraic thinking. Nevertheless, because the present study did not directly measure transfer or analyticity, these findings should not be interpreted as evidence that students failed to transfer knowledge from arithmetic to algebra. Rather, they indicate that the use of structurally related mathematical relationships across arithmetic and algebraic tasks is not automatic.

The interview findings were used to illuminate individual variation in strategy use rather than to reproduce the quantitative performance distribution. Accordingly, students’ explanations were interpreted as explanatory evidence about how they recognized, justified, or revised mathematical relationships across tasks. Cross-task references made during the interviews were interpreted cautiously as evidence that a student perceived a connection between tasks, not as direct evidence of transfer or of an order/priming effect. In this sense, the qualitative data were used to explain how some of the descriptive patterns observed in the written responses were manifested in students’ reasoning.

The findings related to the second research question showed that procedural/computational strategies were dominant at both grade levels, whereas relational/structural strategies varied considerably according to task type and grade level. Relational/structural strategies accounted for only 0.9% of arithmetic and 4.0% of algebraic responses in Grade 6, compared with 5.0% and 26.7%, respectively, in Grade 7. Particularly in Grade 7 algebraic tasks, relational/structural approaches were observed at considerably higher rates, while arithmetic solutions remained predominantly procedural/computational. This quantitative pattern was also reflected in the interview data. In particular, some Grade 7 students explicitly referred to equivalence, common variables, compensation, and relationships among quantities when explaining their solutions, illustrating how the higher proportion of relational/structural strategies observed in the written algebraic tasks was manifested in students’ reasoning. This pattern is consistent with Pitta-Pantazi et al. (2025), who also found that students frequently approached arithmetic expressions as instructions for carrying out calculations rather than as structures reflecting general mathematical properties.

An important implication of this finding is that performance and strategy use did not follow the same pattern. Although students achieved higher correct-response rates in arithmetic tasks, they generally did so through procedural/computational strategies. Thus, a correct arithmetic answer did not necessarily indicate explicit recognition or use of the underlying mathematical structure. The interview findings further supported this interpretation, as some students who produced correct arithmetic solutions explained them primarily through calculation rather than through the mathematical relationship embedded in the task. This interpretation is consistent with Linchevski and Livneh (1999), Subramaniam and Banerjee (2011), and Pittalis (2023), who emphasize that numerical contexts may permit successful computation without requiring students to make structural relationships explicit.

Conversely, relational/structural strategies were more frequent in Grade 7 algebraic tasks despite lower overall performance in those tasks. This should not be interpreted as evidence that letter-symbolic representations themselves produce structural thinking. Rather, when direct numerical computation is less available, some students may attend more explicitly to common variables, equivalence, balancing, and relationships among quantities. The interview data provide examples of this tendency, as some students justified their algebraic solutions by referring directly to common variables or equivalent forms even when their final responses were incomplete or incorrect. Pitta-Pantazi et al. (2025) similarly argue that successful algebraic reasoning may require structural awareness to operate together with analyticity.

Another important finding was that relational/structural strategy use did not always lead to a correct solution. This was particularly visible in Grade 7 algebraic tasks. For example, in C5, 25.0% of students used a relational/structural strategy, yet only 6.7% reached a correct answer, while 18.3% produced an incorrect response. Similar, though smaller, patterns occurred in C2 and C3. The interview findings helped clarify this pattern by showing that some students could identify a relevant mathematical relationship but had difficulty carrying that relationship through to a complete and correct algebraic solution. Thus, the qualitative data provided an explanatory account of why relational/structural strategy use and correct performance did not always coincide. These findings suggest that recognizing and using a mathematical relationship as the basis of a solution is distinguishable from successfully completing the algebraic reasoning required by the task. This interpretation is compatible with Kieran and Martínez-Hernández’s (2022) account of movement from computational toward structural approaches and with Pitta-Pantazi et al.’s (2025) emphasis on the joint role of structural awareness and analytical reasoning. Importantly, the present study did not measure structure sense as a stable psychometric

construct; therefore, these interpretations refer specifically to the relational/structural strategies evident in students' solutions and explanations.

The interview data further illustrated the task-dependent nature of students' strategy use. Miray's solutions were predominantly procedural, although relational reasoning appeared in some tasks and remained tentative in others. Aleyna showed a mixed profile, using procedural strategies in some tasks while drawing directly on mathematical relationships in others. Zeynep, in contrast, used relational/structural approaches more consistently across arithmetic and algebraic tasks, including compensation, equivalence, number properties, and relationships involving variables. These individual profiles provide a qualitative counterpart to the quantitative finding that strategy use varied across tasks and grade levels. Taken together, these findings indicate that students should not be viewed as belonging to fixed "procedural" or "relational" categories. Rather, strategy use may vary across students, tasks, and representations. This interpretation is consistent with Pittalis's (2023) identification of different profiles of arithmetic–algebraic structure sense.

When the findings from the two research questions are considered together, the central contribution of the study becomes clearer: task performance and solution strategy provide related but distinct information about students' mathematical thinking. Students could successfully solve arithmetic tasks through procedural computation without explicitly using the underlying structure, whereas in algebraic tasks some students adopted relational/structural approaches without reaching a correct and complete solution. The interview findings further clarified these contrasting patterns by showing how students could arrive at correct answers through calculation alone, or recognize a mathematical relationship without being able to complete the algebraic reasoning successfully. Accordingly, a correct response alone cannot be treated as evidence of structural reasoning, just as a relational/structural orientation cannot be assumed to guarantee algebraic success.

Overall, the findings support the view that the relationship between arithmetic and algebra cannot be adequately understood solely through performance outcomes. Subramaniam and Banerjee (2011) argue that structural awareness developed in arithmetic cannot be assumed to transfer automatically to algebra, while Kieran (2022) emphasizes the interaction of structural and analytical dimensions in algebraic thinking. Consistent with these perspectives, the present study shows the value of examining not only whether students solve structurally related arithmetic and algebraic tasks correctly, but also how they interpret and use the mathematical relationships embedded in those tasks. By considering written performance data together with interview-based explanations, the study provides a more differentiated account of how students' success, errors, and strategy use may converge or diverge across arithmetic and algebraic contexts.

5. Conclusions and Recommendations

Within the present sample, seventh-grade students showed higher correct-response rates than sixth-grade students on both arithmetic and algebraic tasks. At both grade levels, arithmetic tasks were answered correctly at higher rates than their structurally matched algebraic counterparts. However, the magnitude of this difference varied across task pairs, highlighting the importance of considering the mathematical relationships and task demands involved. Since some matched pairs also differed in task demand, the observed performance differences cannot be attributed solely to representation. These findings describe variation across the participating grade groups and the particular tasks examined.

In terms of solution strategies, procedural/computational approaches predominated at both grade levels, particularly in arithmetic tasks. Relational/structural strategies were observed more frequently in the seventh-grade group's algebraic tasks, although correct-response rates remained higher in arithmetic tasks. Thus, higher correct-response rates and more frequent relational/structural strategy use emerged in different task contexts. This pattern highlights the distinction between obtaining a correct answer and explicitly drawing on the mathematical relationships underlying a task.

A central finding was that correct performance and relational/structural strategy use did not necessarily coincide. Students could obtain correct arithmetic answers by calculating and comparing results without explicitly using the underlying mathematical structure. Conversely, some students recognized and used relevant relationships in algebraic tasks but did not reach complete and correct solutions. Consequently, an incorrect algebraic response did not always indicate an absence of relational reasoning, just as a correct arithmetic response did not necessarily demonstrate explicit structural reasoning. Examining the reasoning underlying a response therefore provided information that its correctness alone could not convey.

The interview findings further illustrated variation in students' solution processes. Among the interviewed students, some relied predominantly on computation, some combined procedural and relational approaches across tasks, and others used relational/structural strategies more consistently. In certain explanations, recognition of equivalence, common quantities, or number properties remained partial or tentative. These findings suggest that students' observed strategy use should be interpreted in relation to the particular task and

the explanation provided. The interviews added explanatory detail by illustrating how similar response outcomes could arise from different reasoning processes and how an individual student's approach could vary across tasks.

Taken together, the findings indicate that performance and strategy use provide related but distinct information about students' mathematical thinking. The study contributes to understanding the relationship between arithmetic and algebra by examining how students interpret and use mathematical relationships across structurally matched tasks. Considering written responses alongside solution strategies and interview explanations makes both successful computation and incomplete relational reasoning more visible. This combined perspective provides a more differentiated account of students' approaches to arithmetic and algebra and highlights the value of attending to their reasoning as well as their final answers.

From an educational perspective, the findings suggest that instruction and assessment should attend not only to whether students obtain correct answers, but also to how they reason about number properties, operations, equivalence, and relationships involving variables. Structurally matched arithmetic and algebraic tasks may be particularly useful for making students' procedural/computational and relational/structural approaches visible and for supporting discussion of how related mathematical relationships are expressed across different forms.

Future research could examine students' strategy use in arithmetic and algebraic tasks longitudinally to explore changes over time. Similar studies could also be conducted across different grade levels, with larger and more diverse samples, and across a broader range of mathematical structures. Qualitative and mixed-methods studies may provide further insight into the conditions under which students adopt relational/structural approaches and the factors that influence whether these approaches lead to complete and correct solutions.

6. Limitations

Several limitations should be considered when interpreting the findings. The quantitative study group was selected through convenience sampling and the comparisons between Grade 6 and Grade 7 were cross-sectional; therefore, the observed grade-level differences should not be interpreted as individual development over time. In addition, the arithmetic and algebraic task pairs were matched on a theory-based structural basis rather than treated as psychometrically equivalent forms, and some pairs differed in their specific response demands. Accordingly, differences between paired tasks cannot be attributed solely to arithmetic versus algebraic notation. The study was also not designed to estimate possible task-order or priming effects. Finally, the interview participants were purposively selected to provide explanatory variation in solution processes, with verbal expressiveness and teacher recommendation among the selection criteria; therefore, the interview findings should be interpreted as explanatory qualitative evidence rather than as representative estimates of the full sample. In addition, although ambiguous and selected responses were reviewed with a second researcher/field expert to support the consistency of coding decisions, the entire dataset was not independently coded by two coders and no quantitative inter-rater reliability coefficient was calculated. This should be considered a limitation of the study when interpreting the classification of students' responses as procedural/computational or relational/structural.

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Appendix

Types of Task	Arithmetic Tasks	Algebraic Tasks
Properties of Numbers	<p>SORU A1</p>  <p>Bir okul kütüphanesine bağışlanan kitaplar iki farklı arabayla taşınmaktadır. Görselde, her arabada bulunan kutulardaki kitap sayıları verilmiştir.</p> <p>Buna göre, 1. arabadaki toplam kitap sayısı ile 2. arabadaki toplam kitap sayısı eşit midir? Eşit değil midir? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>	<p>SORU C1</p>  <p>Bir okul kütüphanesine bağışlanan kitaplar iki farklı arabayla taşınmaktadır. Görselde, her arabada bulunan kutulardaki kitap sayıları verilmiştir.</p> <p>Bilinmeyen kitap sayısı n olmak üzere, 1. arabadaki toplam kitap sayısı ile 2. arabadaki toplam kitap sayısı her zaman eşit midir? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>
	<p>SORU A2</p>  <p>Bir okul kermesinde 72 masa bulunmaktadır. Her masaya 6 mavi ve 5 bordo balon takılması planlanmaktadır. Toplam balon sayısını bulmak isteyen iki öğrenci farklı yollar kullanmıştır.</p> <p>Deniz: $72 \times 6 + 72 \times 5$</p> <p>Ece: $72 \times (6 + 5)$</p> <p>Buna göre Deniz ve Ece'nin kullandığı işlemler aynı toplam balon sayısını verir mi? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>	<p>SORU C2</p>  <p>Bir okul kermesinde süsleme yapmak amacıyla masalara balon takılması planlanmaktadır. Kermeste kullanılacak toplam masa sayısı bilinmediğinden, bu sayı m değişkeni ile ifade edilmiştir. Her masaya 7 mavi ve 9 bordo balon takılacaktır. Toplam balon sayısını hesaplamak isteyen iki öğrenci şu ifadeleri yazmıştır.</p> <p>Alya: $m \times 7 + m \times 9$</p> <p>Alperen: $m \times 16$</p> <p>Buna göre, Alya ve Alperen'in yazdığı ifadeler her zaman aynı toplam balon sayısını mı gösterir, yoksa farklı mı? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>
Properties of Numbers	<p>SORU A3</p>  <p>Bir toptancıda tükenmez kalem paketler halinde satılmaktadır. Her pakette 45 tükenmez kalem bulunmaktadır. Bir kırtasiye dükkânı bu toptancıdan 18 paket mavi ve 12 paket siyah tükenmez kalem sipariş etmiştir.</p> <p>Buna göre kırtasiye dükkânının sipariş ettiği toplam kalem sayısı tek mi, çift mi olur? Ayrıca toplam kalem sayısı 10'un katı mıdır? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>	<p>SORU C3</p>  <p>Bir okulda öğrenciler kavunlu fındıklı paketlemek için Kırmızı Grup ve Beyaz Grup olmak üzere iki gruba ayrılmıştır. Her pakette 50 fındık bulunmaktadır. Kırmızı Grup m paket, Beyaz Grup ise Kırmızı Gruptan 3 paket fazla fındık hazırlamıştır.</p> <p>Buna göre hazırlanan toplam fındık sayısı tek mi, çift mi olur? Ayrıca toplam fındık sayısı 10'un katı olur mu? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>
Properties Related to Equality	<p>SORU A4</p>  <p>Tahterevalli'nin sol tarafında 78 kg ağırlığında bir kişi ve ağırlığı bilinmeyen bir kişi oturmaktadır. Sağ tarafında ise 81 kg ve 24 kg ağırlığında iki kişi bulunmaktadır.</p> <p>Tahterevalli dengede olduğuna göre, ağırlığı bilinmeyen kişi kaç kilogramdır? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>	<p>SORU C4</p>  <p>Tahterevalli'nin sol tarafında iki kişi oturmaktadır. Bu kişilerden birinin ağırlığı $n + 48$ kg, diğeri'nin ağırlığı ise bilinmemektedir. Tahterevalli'nin sağ tarafında da iki kişi oturmaktadır. Bu kişilerden birinin ağırlığı $n + 51$ kg, diğeri'nin ağırlığı ise 21 kg'dır.</p> <p>Tahterevalli dengede olduğuna göre, ağırlığı bilinmeyen kişi kaç kilogramdır? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>
Properties of Operations	<p>SORU A5</p>  <p>Bir otobüsün hareket ettiği otopark ile varacağı otopark arasındaki uzaklık 420 km'dir. Otobüs önce 65 km giderek A durağına ulaşmıştır. Daha sonra B durağına uğramadan yoluna devam etmiş ve 90 km daha giderek C durağına varmıştır. Şoför, B durağındaki yolcuları almayı unuttuğunu fark etmiştir. Bunun üzerine C durağından 65 km geri dönerek B durağına gitmiştir.</p> <p>Buna göre otobüs B durağından itibaren, varacağı otoparka ulaşması için kaç kilometre yolu kalmıştır? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>	<p>SORU C5</p>  <p>Bir otobüsün varacağı otoparka ulaşması için toplam n km yol gitmesi gerekmektedir. Otobüs önce 61 km giderek A durağına ulaşmıştır. Daha sonra B durağına uğramadan, 86 km daha giderek C durağına ulaşmıştır. Şoför daha sonra B durağındaki yolcuları almayı unuttuğunu fark etmiş ve C durağından 61 km geri dönerek B durağına gitmiştir.</p> <p>Buna göre otobüs B durağından itibaren, varacağı otoparka ulaşması için kaç kilometre yolu kalmıştır? [Çözümünüz açıklayınız. Nasıl düşündüğünüzü anlatınız?]</p>